

JOHN-UGWU NNEOMA FAVOUR
18/ENG 02/052 (CARRY OVER)
COMPUTER ENGINEERING

11. If $A = 2i - j$, $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$
Find

- $-3A + 7B - 8C$
- If $K = 2A + 4B - C$ find the direction cosine of K
- $A \times C$ and $B \times C$
- $(3A \times B) \cdot (CA \times 2B)$
- $A - 2B - C$

12. Define perpendicular and coplanar vectors.

Soln

$$\begin{aligned} \text{(i)} \quad & -3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k) \\ & = (-6i + 3j) + (21i + 7j - 77k) - (32i + 32j - 40k) \\ & = (15i + 10j - 77k) - (32i + 32j - 40k) \\ & = -17i - 22j - 37k \end{aligned}$$

$$\text{(ii)} \quad K = 2A + 4B - C$$

$$\begin{aligned} |K| &= \sqrt{2(2i - j) + 4(3i + j - 11k) - 4i + 4j - 5k} \\ &= \sqrt{(4i - 2j) + (12i + 4j - 44k) - 4i + 4j - 5k} \\ &= \sqrt{(16i + 2j - 44k) - (4i + 4j - 5k)} \end{aligned}$$

$$\begin{aligned} \therefore |K| &= \sqrt{12i + 2j - 39k} \\ &= \sqrt{12^2 + 2^2 + 39^2} \\ &= \sqrt{144 + 4 + 1521} \\ &= 37.16 \end{aligned}$$

$$L = \cos \alpha = \frac{12}{37.16}$$

$$M = \cos \beta = \frac{2}{37.16}$$

$$N = \cos \gamma = \frac{39}{37.16}$$

$$\text{(iii)} \quad A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$$

Soln

$$(A \cdot C)B = (2i - j) \cdot (4i + 4j - 5k) (3i + j - 11k)$$

$$(A \cdot B)C = (8i - 4j - 5k) (4i + 4j - 5k)$$

$$\therefore (A \cdot C)B - (A \cdot B)C = (8i - 4j - 5k)(3i + j - 11k) - (8i - 4j - 5k)(4i + 4j - 5k)$$

$$(A \cdot B)C = (8i - 4j - 5k)(4i + 4j - 5k)$$

$$(A \cdot B)C = (8i - 4j - 5k)(4i + 4j - 5k)$$

$$(A \cdot B)C = (8i - 4j - 5k)(4i + 4j - 5k)$$

$$= -6(4i + 4j - 5k)$$

$$= -24i - 2j + 50k$$

$$\therefore (A \cdot C)B - (A \cdot B)C$$

$$= (-3i - j + 11k) - (-24i - 2j + 50k)$$

$$= 21i + j - 19k$$

$$\begin{aligned} \text{(iv)} \quad & [2(2i - j) \times 3(3i + j - 11k)] \cdot [2(2i - j) \times 2(3i + j - 11k)] \\ &= [6i - 3j \times 3i + 7j - 11k] \cdot [2(2i - j) \times 6i + 2j - 22k] \\ &= (18i - 3j - 11k) \cdot (12i - 2j - 22k) \\ &= (216i + 6j + 242k) \\ &= 216i + 6j + 242k \end{aligned}$$

$$\text{(v)} \quad A - 2B - C$$

$$= 2i - j - 2(3i + j - 11k) - 4i + 4j - 5k$$

$$= (2i - j) - (6i - 2j - 22k) - (4i + 4j - 5k)$$

$$= -8i - 3j - 17k$$

Perpendicular Vectors; if $A \cdot B = 0$

Coplanar Vectors: Vectors $\vec{A}, \vec{B}, \vec{C}$ are coplanar if they are parallel and lie on the same plane. (ie $\vec{A}(\vec{B} \times \vec{C}) = 0$)

(A x B)

$$\begin{aligned} & 1 \quad 2 \quad 5 \\ & \begin{vmatrix} 1 & 2 & 5 \\ 2 & 5 & -1 \\ 5 & -1 & 2 \end{vmatrix} = 1(1 \times 2 - 5 \times 2) - 2(1 \times 2 - 5 \times 2) + 5(1 \times 2 - 5 \times 2) \\ & = -18i + 5j - k \end{aligned}$$