MAT 104 (ASSIGNMENT 8)

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Examine whether or not these pairs of lines are perpendicular to each other.

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

For perpendicular lines, the product of the slopes, $m_1 \times m_2 = -1$

$y - 3x - 2 = 0$

$y = 3x + 2$, $\boxed{m_1 \Rightarrow 3}$

$3y + x + 9 = 0$

$\frac{3y}{3} = \frac{-9 - x}{3}$, $y = \frac{-9 - x}{3}$

$y = \frac{-9}{3} - \frac{x}{3} = -3 - \frac{x}{3}$, $\boxed{m_2 \Rightarrow -1/3}$

$\boxed{m_1 \Rightarrow 3 \text{ and } m_2 \Rightarrow -1/3}$

$m_1 \times m_2 = 3 \times -\frac{1}{3}$ $\boxed{\Rightarrow -1}$

Since $m_1 \times m_2 \Rightarrow -1$ hence the pairs of lines are perpendicular to each other.

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

For perpendicular lines, the product of their slopes, $m_1 \times m_2 = -1$

$3y - 4 = 2x + 3$

$3y = 2x + 3 + 4$, $\frac{3y}{3} = \frac{2x + 7}{3}$

$y = \frac{2x}{3} + \frac{7}{3}$, $\boxed{m_1 \Rightarrow 2/3}$

$y - 5 = x + 6$

$y = x + 6 + 5$, $y = x + 11$

$\boxed{m_2 \Rightarrow 1}$

$\boxed{m_1 \Rightarrow 2/3 \text{ and } m_2 \Rightarrow 1}$

$m_1 \times m_2 = \frac{2}{3} \times 1$ $\boxed{\Rightarrow \frac{2}{3}}$

Since $m_1 \times m_2 \neq -1$ hence the pairs of lines are not perpendicular to each other.

3) $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

(i) Equation of tangent

$m = 2x + 2y\, dy/dx + 3x \cdot 1\, dy/dx + 3y = 0$

$2y\, dy/dx + 3x\, dy/dx = -3y - 2x$

$dy/dx\,(2y + 3x) = -3y - 2x$

$\frac{2y+3x}{2y+3x}$ $\qquad$ $2y+3x$

$dy/dx = \frac{-3y-2x}{2y+3x}$

$m = \frac{-3(2) - 2(1)}{2(2) + 3(1)} = \frac{-6-2}{4+3} = \frac{-8}{7}$,,

$\therefore m = -8/7$

$m = -8/7, x_1 = 1, y_1 = 2$

Equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - 2 = -\frac{8}{7}(x-1)$, $7(y-2) = -8(x-1)$

$7y - 14 = -8x + 8$

$7y + 8x - 14 - 8 = 0$, $7y + 8x - 22 = 0$

$\therefore$ Equation of tangent $\Rightarrow 7y + 8x - 22 = 0$

(ii) Equation of normal

for normal, $m_1 \times m_2 = -1$, $m_1 = -8/7$

$\frac{-8/7}{-8/7} \times m_2 = \frac{-1}{-8/7}$, $m_2 = -1 \div \frac{-8}{7}$, $-1 \times \frac{7}{8} = \frac{7}{8}$ $\quad \therefore m_2 \Rightarrow 7/8$

$m = 7/8, x_1 = 1, y_1 = 2$

Equation of normal $\Rightarrow y - y_1 = m(x - x_1)$

$y - 2 = \frac{7}{8}(x-1)$, $8(y-2) = 7(x-1)$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$, $8y - 7x - 9 = 0$

$\therefore$ Equation of normal $\Rightarrow 8y - 7x - 9 = 0$