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MAT 104 ASSIGNMENT

1. Find the limit of the function $\frac{(x - \cos x)}{x}$ as $x \to 0$

2. If $y = -3\tan 7x e^{3x}$, find dy/dx

3. If $y = \cos 3x$, Find dy/dx from the first principle

4. Given that $f(x) = 2x^3 - 7x$ and $g(x) = -3x$. find $(f-g)(5)$

5. Find $f \circ g(x)$ if $f(x) = 4x^2 + 2$ and $g(x) = 2x + 3$

6. Find the gradient of $x^2 + 2xy + y^2 = 1{,}020$

7. Find the first derivative of the function $y = x^2 \cos x$

Soln

1. $$\frac{(x - \cos x)}{x}$$

By L'Hopital's Rule, we have

$$\lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = 1 + \sin 0 = 1 + 0 = 1$$

2. $y = -3\tan 7x e^{3x}$

$$\frac{dy}{dx} = -3(7\sec^2 7x) 3e^{3x}$$

$$= -21 \sec^2 7x (3e^{3x})$$

$$= -63 \sec^2 7x e^{3x}$$

3. If $y = \cos 3x$, find dy/dx from the first principle

Soln

$$\frac{dy}{dx} \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\cos[3(x+\Delta x)] - \cos 3x}{\Delta x}$$

$$= \frac{\cos[3x + 3\Delta x] - \cos 3x}{\Delta x}$$

Recall,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{[\cos 3x \cos(3\Delta x) - \sin 3x \sin(3\Delta x)] - \cos 3x}{\Delta x}$$

$$= \left[\left(\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right)\right] - \left[\left(\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right)\right]$$

$$= \lim_{\Delta x \to 0} \left[\frac{\cos 3x \cos(3\Delta x) - \cos 3x}{\Delta x}\right] - \left[\frac{\sin 3x \sin(3\Delta x)}{\Delta x}\right]$$

$$= \lim_{\Delta x \to 0} \left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\Delta x}\right)\right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x}\right)\right]$$

$$\cancel{= \lim_{\Delta x \to 0} \left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\cos(3\Delta x + 1)\Delta x}\right)\right]}$$

$$= \lim_{\Delta x \to 0} \left[\cos 3x \left(\frac{\cos(3\Delta x) - 1}{\Delta x} \times \frac{\cos(3\Delta x + 1)}{\cos(3\Delta x + 1)} \right) \right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right]$$

$$= \lim_{\Delta x \to 0} \left[\cos 3x \left(\frac{\cos^2(3\Delta x) - 1}{(\cos(3\Delta x + 1))\Delta x} \right) - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right] \right]$$

from the identity $\sin^2(\theta) + \cos^2(\theta) = 1$

$$\cos^2(3\Delta x) - 1 = -\sin^2(3\Delta x)$$

$$= \lim_{\Delta x \to 0} \left[-\cos 3x \left(\frac{\sin^2(3\Delta x)}{(\cos(3\Delta x + 1))\Delta x} \right) \right] - \left[\sin 3x \left(\frac{\sin(3\Delta x)}{\Delta x} \right) \right]$$

$$= \lim_{\Delta x \to 0} \left[\frac{-\cos 3x \sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x} - \sin 3x \right] \left[\frac{\sin(3\Delta x)}{\Delta x} \right]$$

$$= \left(\lim_{\Delta x \to 0} \left[\frac{-\cos 3x \sin^2(3\Delta x)}{\cos(3\Delta x + 1)\Delta x} - \sin 3x \right] \right) - \left(\lim_{\Delta x \to 0} \left[\frac{\sin(3\Delta x)}{\Delta x} \right] \right)$$

First limit $\Rightarrow$ $-\dfrac{(\cos 3x \sin^2(3(0)))}{\cos(0+1)\,0} - \sin 3x$

Rewritten as $\Rightarrow$ $-\sin 3x \lim_{\Delta x \to 0} \dfrac{\sin(3\Delta x)}{\Delta x}$

Recall $\lim_{\Delta x \to 0} \dfrac{\sin x}{x} = 1$, therefore $\lim_{\Delta x \to 0} \dfrac{\sin(3\Delta x)}{\Delta x} = 3$

Final form ends up a $(-\sin 3x)(3)$

$$= -\sin 3x$$

4. $f(x) = \cancel{2x^3} \; 2x^3 - 7x$, $g(x) = -3x$

$= (f-g)(5)$

$(f-g)(5) = [2(5)^3 - 7(5)] - (-3(5))$

$= [2(125) - 35] - (-15)$

$= 250 - 35 + 15$

$= 230$

6. $x^2 + 2xy + y^2 = 1{,}020$

Solution

$x^2 + 2xy + y^2 = 1{,}020$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$

$\frac{dy}{dx}(2x + 2y) = -2x - 2y$

$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$

7. First derivative of the function $y = x^2 \cos x$

Soln.

$y = x^2 \cos x$

$u = x^2$, $v = \cos x$

$\frac{du}{dx} = 2x$, $\frac{dv}{dx} = -\sin x$.

Using product rule

$$\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2)(-\sin x) + (2x)(\cos x)$$

$$= -x^2 \sin x + 2x\cos x$$

$$= [2x\cos x - x^2 \sin x]$$

5. $fog = 4(2x+3)^2 + 2$

$$= 4[(2x+3)(2x+3)] + 2$$

$$= 4(4x^2 + 6x + 6x + 9) + 2$$

$$= 4(4x^2 + 12x + 9) + 2$$

$$= 16x^2 + 48x + 36 + 2$$

$$= 16x^2 + 48x + 88$$

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