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QUESTION: If $A = 2i - j$, $B = 3i + j - 11K$ and $C = 4i + 4j - 5K$, find

i) $-3A + 7B - 8C$

ii) If $K = 2A + 4B - C$, find the direction cosine of K

iii) $A \times (B \times C)$

iv) $(3A \times B) \cdot (A \times 2B)$

v) $A - 2B - C$

SOLUTION

i) $-3A + 7B - 8C$

where $-3A = -3(2i - j) = -6i + 3j$

$7B = 7(3i + j - 11K) = 21i + 7j - 77K$

$-8C = -8(4i + 4j - 5K) = -32i - 32j + 40K$

$\therefore -3A + 7B - 8$

$= (-6i + 3j) + (21i + 7j - 77K) + (-32i - 32j + 40K)$

$= (-6 + 21 - 32)i + (3 + 7 - 32)j + (-77 + 40)K$

$= 17i - 22j - 37K$

ii) If $K = 2A + 4B - C$, find the direction cosine of K

where $2A = 2(2i - j) = 4i - 2j$

$4B = 4(3i + j - 11K) = 12i + 4j - 44K$

$-C = -(4i + 4j - 5K) = -4i - 4j + 5K$

$K = 2A + 4B - C$

$= (4i - 2j) + (12i + 4j - 44K) + (-4i - 4j + 5K)$

$= (4 + 12 - 4)i + (-2 + 4 - 4)j + (-44 + 5)K$

$= 12i - 2j - 39K$

$|K| = \sqrt{(12)^2 + (-2)^2 + (-39)^2}$

$= \sqrt{144 + 4 + 1521}$

$= \sqrt{1,669}$

$= 40.85$

$\text{Cos}K = \dfrac{12i - 2j - 39K}{40.85}$

$\therefore l = \text{Cos}\alpha = 12/40.85$

$m = \text{Cos}\beta = -2/40.85$

$n = \text{Cos}\gamma = -39/40.85$

iii) $A \times (B \times C)$

$B \times C = \begin{vmatrix} i & j & K \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$

$= i\begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - j\begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + K\begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$

$= i(-5 + 44) - j(-15 + 44) + K(12 - 4)$

$= 39i - 29j + 8k$

$$\therefore A \times (B \times C) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 39 & -29 \end{vmatrix}$$

$$= i(-8+0) - j(16-0) + k(-58+39)$$

$$= -8i - 16j - 19k$$

iv) $(3A \times B) \cdot (A \times 2B)$

Where $A = 2i - j$

$3A = 3(2i - j) = 6i - 3j$

$B = 3i + j - 11k$

$2B = 2(3i + j - 11k) = 6i + 2j - 22k$

$$(3A \times B) = \begin{vmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$= i\begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - j\begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + k\begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$$i(33-0) - j(-66-0) + k(6+9)$$

$$= 33i + 66j + 15k$$

$$(A \times 2B) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$= i(22-0) - j(-44-0) + k(4+6)$$

$$= 22i + 44j + 10k$$

$\therefore (3A \times B) \cdot (A \times 2B)$

$$= (33i + 66j + 15k) \cdot (22i + 44j + 10k)$$

$$= 726 + 2904 + 150$$

$$= 3,708 \text{ Ω}$$

v) $A - 2B - C$

$-2B = -2(3i + j - 11k)$

$= -6i - 2j + 22k$

$-C = -(4i + 4j - 5k) = -4i - 4j + 5k$

$A - 2B - C = (2i - j) + (-6i - 2j + 22k) + (-4i - 4j + 5k)$

$= (2 - 6 - 4)i + (-1 - 2 - 4)j + (22 + 5)k$

$= -8i - 7j + 27k$ Ω

(2) Define Perpendicular and Co-planar Vectors

PERPENDICULAR VECTORS are said to be two vector A & B, whereby their dot product is equal to zero. i.e $(A \cdot B) = 0$

CO-PLANAR VECTORS are three vectors A, B and C, if $A \cdot (B \times C) = 0$.