

Question: Examine whether or not these pair of lines are perpendicular to each other. 1, $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

$$2, 3y - 4 = 2x + 3 \text{ and } y - 5 = 5x + 6$$

3, Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

Solution

$$1, y - 3x - 2 = 0 \text{ and } 3y + x + 9 = 0$$

$$\text{let } A = y - 3x - 2 = 0$$

$$= \frac{dy}{dx} - 3 - 0 = 0$$

$$= \frac{dy}{dx} = 3 = 0$$

$$= \frac{dy}{dx} = 3$$

$$\text{let } B = 3y + x + 9 = 0$$

$$3 \frac{dy}{dx} + 1 + 0 = 0$$

$$3 \frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -\frac{1}{3}$$

$$A + B$$

$\therefore y - 3x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

$$2. \quad 3y - 4 = 2x + 3 \text{ and } y - 5 = x + 6$$

$$\text{let } A - 3y - 4 = 2x + 3$$

$$\frac{3dy}{dx} - 0 = 2 + 0$$

$$\frac{3dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

$$\text{let } b = y - 5 = x + 6$$

$$\frac{dy}{dx} = 0 = 1 + 0$$

$$\frac{dy}{dx} = 1$$

$$A \neq B$$

$\therefore 3y - 4 = 2x + 3$ and $y - 5 = x + 6$ is not perpendicular

$$3x^2 + y^2 + 3y - 11 = 0 \text{ at point } (1, 2)$$

$$2x + 2y \frac{dy}{dx} + 3\left(x \times \frac{dy}{dx} + y \times 1\right) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

when $x = 1$ and $y = 2$

$$m = -[2(1) + 3(2)]$$

$$2(2) + 3(1)$$

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$$= -\frac{(2+6)}{4+3} = -\frac{8}{7}$$

$$\therefore m = -\frac{8}{7}$$

Equation of the tangent to a curve

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$y - 2 = -\frac{8x}{7} + \frac{8}{7}$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$8x + 7y - 22 = 0$ which gives the equation of the tangent

Equation of the normal to a curve

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$y - 2 = \frac{7x}{8} - \frac{7}{8}$$

$$8y - 16 = 7x - 7$$

$$8y = 7x - 7 + 16$$

$7x - 8y + 9 = 0$, which gives the equation of the normal