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Assignment

Examine whether or not these pair of lines are perpendicular to each other.

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

$$y - 3x - 2 = 0$$

$$y = 3x + 2$$

$$y = mx + c$$

$$m_1 = 3$$

$$3y + x + 9 = 0$$

$$3y = -x - 9$$

$$y = -\frac{x}{3} - \frac{9}{3}$$

$$y = -\frac{x}{3} - 3$$

$$y = -\frac{x}{3} - 3$$

$$y = mx + c$$

$$m_2 = -\frac{1}{3}$$

Recall that $m_1 m_2 = -1$ if the lines are perpendicular,

$$3 \times -\frac{1}{3} = -1$$

\therefore The lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular to each other.

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

$$3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x + 7}{3}$$

$$y = mx + c$$

$$m_1 = \frac{2}{3}$$

$$y-5 = x+6$$

$$y = x+6+5$$

$$y = x+11$$

$$y = mx+c$$

$$m_2 = 1$$

Recall that $m_1 m_2 = -1$ if the lines are perpendicular

$$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

\therefore The lines $3y-4 = 2x+3$ and $y-5 = x+6$ are ~~per~~ not perpendicular to each other.

3. Find the equations of the tangent and normal to the Curve:

$$x^2 + y^2 + 3xy - 11 = 0 \text{ at the point } x=1, y=2$$

Solution

$$x^2 + y^2 + 3xy - 11 = 0$$

$$2x + 2y \frac{dy}{dx} + 3(y + x \frac{dy}{dx}) = 0$$

$$2x + 2y \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$2x + 2y \frac{dy}{dx} + 3(1 \cdot y + x \cdot \frac{dy}{dx}) = 0$$

$$2x + 2y \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\frac{dy}{dx} = \frac{-2(1) - 3(2)}{2(2) + 3(1)}$$

$$\frac{dy}{dx} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7} = m$$

$$\frac{dy}{dx} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7} = m$$

The equation of the tangent gives,

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$$7y + 8x - 22 = 0$$

The equation of the normal gives,

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y-2 = \frac{7}{8}(x-1)$$

$$8(y-2) = 7(x-1)$$

$$8y-16 = 7x-7$$

$$8y-7x-16+7=0$$

$$8y-7x-9=0$$