

MAT 104 Assignment

19/MUSO1/407

Examine whether or not these pair of lines are perpendicular to each other.

$$\downarrow \quad Y - 3x - 2 = 0 \quad \text{and} \quad 3y + x + 9 = 0$$

$$2 \quad 3y - 4 = 2x + 3 \quad \text{and} \quad y - 5 = x + 6$$

3 Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

Solve

$$\downarrow \quad y - 3x - 2 = 0$$

$$3y + x + 9 = 0$$

$$\text{Let } A = y - 3x - 2 = 0$$

$$B = 3y + x + 9 = 0$$

$$\frac{dy}{dx} = -3 \quad \text{and} \quad \frac{dy}{dx} = -3 - 0, \quad \frac{dy}{dx} = -3 = 0 \therefore \frac{dy}{dx} = 3$$

differentiating B

$$3 \frac{dy}{dx} + 1 + 0 = 0$$

$$\therefore \frac{dy}{dx} = -\frac{1}{3}$$

$$A + B$$

$$\text{i.e. } y - 2x - 2 = 0 \text{ is perpendicular to } 3y + x + 9 = 0$$

$$2 \quad 3y - 4 = 2x + 3$$

$$y - 5 = x + 6$$

Re arrange

$$3y - 2x - 4 - 3 = 0, \quad 3y - 2x - 7 = 0$$

$$3y - 2x - 5 - 6 = 0, \quad y - x - 11 = 0$$

$$y - x - 5 - 6 = 0, \quad y - x - 11 = 0$$

$$\text{Let } A = 3y - 2x - 7 = 0 \therefore \frac{dy}{dx} = \frac{2}{3}$$

$$\text{Let } B = y - x - 11 = 0$$

$$\frac{dy}{dx} - 1 + 0 = 0$$

$$\therefore \frac{dy}{dx} = 1$$

$\therefore 3y - 4 = 2x + 3$ is not perpendicular to $y - 5 = x + 6$

3, $x^2 + y^2 + 3y - 11 = 0$ at point $(1, 2)$

$$2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \times 1 \right) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y - 0 = 0$$

$$2x + 3y = -2y \frac{dy}{dx} + 3x \frac{dy}{dx}$$

$$2x + 3y = -\frac{dy}{dx} (2y + 3x)$$

$$\frac{dy}{dx} = -\frac{(2x + 3y)}{2y + 3x}$$

$$m = \frac{dy}{dx} = -\frac{(2x + 3y)}{2y + 3x}$$

$$-1 \pm \frac{-8}{7}$$

$$-1 \times \frac{7}{8} =$$

when $x = 1$ and $y = 2$

$$m = -\frac{(2(1) + 3(2))}{2(2) + 3(1)} = -\frac{(2 + 6)}{4 + 3} = -\frac{8}{7}$$

Equation of the tangent to a Curve

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 + 8 = 0$$

$$7y + 8x - 22 = 0$$

Equation of the normal to a Curve

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0$$