

Assignment

Examine whether or not these pairs of lines are perpendicular to each other:-

1, $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Sol \rightarrow

$$y - 3x - 2 = 0$$

$$y = 3x + 2$$

$$[y = mx + c]^*$$

$$\therefore M_1 = 3$$

$$3y + x + 9 = 0$$

$$3y = -x - 9$$

$$y = \frac{-x}{3} - \frac{9}{3}$$

$$\therefore M_2 = -\frac{1}{3}$$

$$\text{Proof } \perp M_1 \times M_2 = 3 \cdot -\frac{1}{3} = -1$$

The Pairs of lines are \therefore perpendicular as

$$3 \cdot -\frac{1}{3} = -1 \quad (y - 3x - 2 = 0 \text{ is perpendicular to } 3y + x + 9 = 0)$$

2, $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Sol \rightarrow

$$3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x}{3} + \frac{7}{3}$$

$$\therefore M_1 = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 6 + 5$$

$$y = x + 11$$

$$\therefore M_2 = 1$$

(Continuation)

MBBS

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The pairs of lines are therefore "not" perpendicular as
 $m_1 \times m_2 \rightarrow \frac{2}{3} \times 1 = \frac{2}{3}$

($3y - 4 = 2x + 3$ is not perpendicular to $y - 5 = x + 6$)

3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

sol \rightarrow

$$2x + 2y \frac{dy}{dx} + 3 \left(x \times \frac{dy}{dx} + y \times 1 \right) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\therefore m = \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

\therefore when $x = 1$ and $y = 2$

(continuation)

$$m = -\frac{2(1) + 3(2)}{2(2) + 3(1)}$$

$$= -\frac{(2+6)}{4+3} = -\frac{8}{7}$$

$$\therefore m = -8/7$$

Equation of the tangent to a curve:-

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$8x + 7y - 22 = 0$$

\(\therefore\) equation of tangent to a curve = $8x + 7y - 22 = 0$

Equation of the normal to a curve:-

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{(-8/7)}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0$$

\(\therefore\) equation of normal to a

curve = $8y - 7x - 9 = 0$