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MBBS
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MTH104

Questions

A Examine whether or not these pairs of lines are perpendicular to each other.

II) $y - 3x - 2 = 0$, and
$3y + x + 9 = 0$.

Soln

For these two lines to be perpendicular to each other then,
$M_1 M_2 = -1$

Find the values of M_1 & M_2 using the equation for line
$Y = mx + c$

For M_1

$y - 3x - 2 = 0$

$y = 3x + 2$

$M_1 = 3$

For M_2

$3y + x + 9 = 0$

$\frac{3y}{3} = \frac{-x}{3} - \frac{9}{3}$

$y = -\frac{1}{3}x - 3$

$M_2 = -\frac{1}{3}$

$M_1 M_2 = -1$

$3 \times -\frac{1}{3} = -1$

∴ The lines
$y - 3x - 2 = 0$, and
$3y + x + 9 = 0$
are perpendicular to each other //

2. Solu

$3y - 4 = 2x + 3$; and

$y - 5 = x + 6$.

M_1, for the equation

$3y = 2x + 3 + 4$

$\frac{3y}{3} = \frac{2x}{3} + \frac{7}{3}$

$y = \frac{2}{3}x + \frac{7}{3}$

$M_1 = \frac{2}{3}$

M_2, second equation

$y = x + 6 + 5$

$y = x + 11$

$M_2 = 1$.

$M_1 M_2 = -1$.

$\frac{2}{3} \times 1 = \frac{2}{3}$

∴ The equation of line $3y - 4 = 2x + 3$ & $y - 5 = x + 6$ are not perpendicular to each other. //

B Find the equations of the tangent and normal to the curve

$x^2 + y^2 + 3xy - 11 = 0$

$(x=1, y=2)$.

Solu

$m = \frac{dy}{dx}$ of $x^2 + y^2 + 3xy - 11 = 0$.

$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$.

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2x + 3y + \frac{dy}{dx}(2y + 3x) = 0$

$\frac{dy}{dx}(2y + 3x) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$

$m = \frac{dy}{dx}\Big|_{1,2} = \frac{-2(1) \mp 3(2)}{2(2)+3(1)} = \frac{-2 \mp 6}{4+3} = \frac{-8}{7}$

$M_1 = -8/7$

The equation of the Tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -8/7(x - 1)$

$y - 2 = -8x/7 + 8/7$

multiply through by 7.

$7y - 14 = -8x + 8$

$7y = -8x + 8 + 14$

$7y = -8x + 22$

$7y + 8x - 22 = 0$ // (Equation of Tangent)

Equation for the Normal

$M_2 = -1 \div -8/7$

$M_2 = -1 \times 7/-8$

$M_2 = 7/8.$

$y - 1 = 7/8(x - 2)$

$y - 1 = 7x/8 - 14/8$

$8y - 8 = 7x - 14$

$8y = 7x - 14 + 8$

$8y = 7x - 6.$ //

$y - 2 = 7/8(x - 1)$

$y - 2 = 7x/8 - 7/8$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$

$8y - 7x - 7 = 0$ (Equation of the Normal)