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Assignment Title: MR OKUNLOLA AND DR OYEELAMI'S GROUPS

Questions:

1. A particle moves along a curve $x = 2t^7$, $y = 6t^2 - 4t$, $z = t - 5$, where t is time. Find its velocity.

Solution

The position vector $r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

$$r = (2t^7)\mathbf{i} + (6t^2 - 4t)\mathbf{j} + (t - 5)\mathbf{k}$$

$$\text{Velocity } v = \frac{dr}{dt}$$

$$\frac{dr}{dt} = 14t + (2t - 4)\mathbf{j} + \mathbf{k}$$

2. If $\vec{A} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$, $\vec{B} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\vec{C} = 4\mathbf{j} - 3\mathbf{k}$, find $\vec{A} \times (\vec{B} \times \vec{C})$

Solution

$$\vec{B} \times \vec{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$\vec{B} \times \vec{C} = \mathbf{i}(9 - 4) - \mathbf{j}(-6 - 0) + \mathbf{k}(5 - 0)$$

$$\vec{B} \times \vec{C} = 5\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$$

Hence:

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -4 \\ 5 & 6 & 5 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -4 \\ 5 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 5 & 5 \end{vmatrix}$$

$$= 1(16 - (-20)) - j(8 - (-20)) + k(5 - 10)$$

$$= 1(16 + 20) - j(8 + 20) + k(-5)$$

$$= 36i - 28j - 5k$$

3. Given $R = 4\sin 3t i + 4e^{2t} j + t^3 k$
Find the Integral of R with respect to t

Solution

$$\frac{dr}{dt} = 12 \cos 2t i + 3e^{2t} j + 2t^2 k$$

4. If $A = 2i + 2j - k$, $B = 2i + j + 4k$, $C = i + j + k$. Find $(A+C) \cdot (B-A)$

Solution

$$(A+C) = (2i + 2j - k) + (i + j + k)$$

$$= 3i + 3j$$

$$(B-A) = (2i + j + 4k) - (2i + 2j - k)$$

$$= 0i - j + 5k$$

$$= 5i - j + 5k$$

$$\therefore (A+C) \cdot (B-A) = (3i + 3j) \cdot (5i - j + 5k)$$

$$= 30 - 3$$

$$= 27$$

5. Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$.

$$r = t_i + t^2_j + t^3_k$$

$$\frac{dr}{dt} = i + 2t_j + 3t^2_k$$

$$\text{at } t=1 = \frac{dr}{dt} = i + 2j + 3k$$

$$\left| \frac{dr}{dt} \right|_{t=1} = \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{1+4+9}$$

$$= \sqrt{14}$$

$$= \cancel{4} 3.7$$

$$\text{Hence } T = \frac{i + 2j + 4k}{3.7}$$