3. $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

$$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} + 3\left(x\cdot\frac{dy}{dx} + y\cdot 1\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx}(2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \left.\frac{dy}{dx}\right|_{x=1,\, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$$

a) Equation of tangent

$$y = mx + c$$

$$2 = \frac{-8}{7}(1) + c$$

$$14 = -8 + 7c$$

$$22 = 7c$$

$$c = 22/7$$

$$y = \frac{-8}{7}x + \frac{22}{7}$$

$7y = -8x + 22$ (Equation of the tangent)

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The lines will be perpendicular if the product of the gradient of the two lines is equals to -1 $(m_1 m_2 = -1)$

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$
Making y the subject of formula
$y = 3x + 2$ and $y = \frac{-x}{3} - 3$

Gradient of the first line $M_1 = 3$
Gradient of the second line $m_2 = -\frac{1}{3}$
Applying formula:
$m_1 m_2 = -1$ $\Rightarrow$ $\cancel{3} \times \frac{-1}{\cancel{3}} = -1$ ∴ The pairs of line are perpendicular to each other.

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$
Making y the subject formula
$y = \frac{2x}{3} + \frac{7}{3}$ and $y = x + 11$

Gradient of the first line $m_1 = \frac{2}{3}$

Gradient of the second line $m_2 = 1$
Applying formula:
$m_1 m_2 = -1$ $= \frac{2}{3} \times 1 \neq -1$ ∴ The pair of lines are not perpendicular to each other

b) Equation of normal

$$y = mx + c$$

~~x = -8~~

b) Equation of normal

$$m_1 m_2 = -1$$

$$\frac{-8}{7} \times m_2 = -1$$

$$m_2 = -1 \div \frac{-8}{7}$$

$$m_2 = \cancel{-}1 \times \frac{-7}{\cancel{-}8} = \frac{7}{8}$$

Applying formula

$$y = mx + c$$

$$2 = \frac{7}{8}(1) + c$$

$$2 = \frac{7}{8} + c$$

$$16 = 7 + 8c$$

$$9 = 8c$$

$$c = 9/8$$

$$y = \frac{7x}{8} + \frac{9}{8}$$

$8y = 7x + 9$ (Equation of normal)