PINNICK TISE ORITSETSERUNDEDE

CHEMICAL ENGINEERING

19/ENG01/013

Serial Number: 48

Math 102 Assignment

Question 1

If $A = 2i - j$; $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$, find the following

i $-3A + 7B - 8C$

ii If $K = 2A + 4B - C$ find the direction cosine of K.

iii $(A \times B \times C)$

iv $(3A \times B) \cdot (A \times 2B)$

v $A - 2B - C$

Solution

i $-3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k)$

$-6i + 3j + 21i + 7j - 77k - 32i - 32j + 40k$

$= -6i + 21i - 32i + 3j + 7j - 32j - 77k + 40k$

$= -17i - 22j - 37k$

$\therefore -3A + 7B - 8C = -17i - 22j - 37k$

ii

ii $K = 2A + 4B - C = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$

$K = 4i - 2j + 12i + 4j - 44k - 4i - 4j + 5k$

$K = 4i + 12i - 4i - 2j + 4j - 4j - 44k + 5k$

$K = 12i - 2j - 39k$

$|K| = \sqrt{12^2 + (-2)^2 + (-39)^2} = \sqrt{144 + 4 + 1521} = \sqrt{1669} = 40.85$

Direction cosine of K;

$l = \cos\alpha = 12/40.85 = 0.29$

$M = \cos\beta = -2/40.85 = -0.05$

$N = \cos\gamma = -39/40.85 = -0.95$

(iii) $\begin{vmatrix} i & j & k \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$ $= B \times C$

$i\begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - j\begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$

$i(-5+44) - j(-15+44) + k(12-4)$

$39i - 29j + 8k = B \times C$

$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$

$i\begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 39 & -29 \end{vmatrix}$

$i(-8-0) - j(16-0) + k(-58+39)$

$= -8i - 16j - 19k$ //

iv $3A = 3(2i - j) = 6i - 3j$

$$3A \times B = \begin{vmatrix} i & j & K \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$i\begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - j\begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + K\begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$= i(33 - 0) - j(-66 - 0) + K(6 + 9)$

$= 33i + 66j + 15K = (3A \times B)$

$2B = 2(3i + j - 11K) = 6i + 2j - 22K$

$$A \times 2B = \begin{vmatrix} i & j & K \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$i\begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + K\begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$= i(22 - 0) - j(-44 - 0) + K(4 + 6)$

$= 22i + 44j + 10K = (A \times 2B)$

$\therefore (3A \times B) \cdot (A \times 2B) = (33 + 66j + 15K) \cdot (22i + 44j + 10K)$

$(726) + (2904) + (50)$

$726 + 2904 + 50$

$\therefore (3A \times B) \cdot (A \times 2B) = 3680$ u

v $A - 2A - C = (2i - j) - 2(3i + j - 11K) - (4i + 4j - 5K)$

$= 2i - j - 6i - 2j + 22K - 4i - 4j + 5K$

$= 2i - 6i - 4i - j - 2j - 4j + 22K + 5K$

$\therefore A - 2A - C = -8i - 7j + 27K$ u

Question 2

Define perpendicular and co-planar vectors.

Solution

Perpendicular Vectors:

Two vectors A and B are said to be perpendicular if $\bar{A} \cdot \bar{B} = 0$

Co-planar Vectors:

Three vectors A, B and C are said to be co-planar if $\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$