

ERADAJAYE VICTOR MUDIAGA
MAT104 Assignment

19/MHS01/156

Examine whether or not these pairs of lines are perpendicular to each other

1 $y - 3x - 2 = 0$ & $3y + x + 9 = 0$

2 $3y - 4 = 2x + 3$ & $y - 5 = x + 6$

3 Find the equation to the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution

1 ~~$y =$~~ $y - 3x - 2 = 0$ & $3y + x + 9 = 0$

Let ~~$y =$~~ $y = 3x + 2$

$$\frac{dy}{dx} = 3$$

$$\therefore m_1 = 3$$

$$3y = -x - 9$$

$$3 \frac{dy}{dx} = -1$$

$$3m_2 = -1$$

$$m_2 = -\frac{1}{3}$$

Since $m_1 = \frac{-1}{m_2}$ Hence they are perpendicular

$$2 \quad 3y - 4 = 2x + 3$$

$$y - 5 = x + 6$$

$$3y = 4 + 2x + 3$$

$$3 \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

$$\therefore m_1 = \frac{2}{3}$$

$$y = x + 6 + 5$$

$$\frac{dy}{dx} = 1$$

$$m_2 = 1$$

Since $m_1 \neq -\frac{1}{m_2}$ These lines are not perpendicular

3

$$x^2 + y^2 + 3xy - 11 = 0 \text{ at point } (1, 2)$$

$$2x + 2y + 3(x \frac{dy}{dx} + y) - 0 = 0$$

$$2x + 2y + 3x \frac{dy}{dx} + 3y = 0$$

$$3 \quad x^2 + y^2 + 3xy - 11 = 0 \quad \text{at point } (1, 2)$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y - 0 = 0$$

$$2x + 3x \frac{dy}{dx} + 3y + 2y \frac{dy}{dx} = 0$$

$$3x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 2y}$$

$$\text{where } x=1 \quad y=2$$

$$\frac{dy}{dx} = \frac{-2(1) - 3(2)}{3(1) + 2(2)} = \frac{-2 - 6}{3 + 4} = \frac{-8}{7}$$

$$m = -\frac{8}{7} //$$

For Equation of the tangent

$$(y - y_1) = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y = -8x + 22$$

$$y = -\frac{8}{7}x + \frac{22}{7} //$$

For Equation of the Normal

$$(y - y_1) = -\frac{1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{-8/7}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y = 7x + 9$$

$$y = \frac{7x}{8} + \frac{9}{8}$$

$$y = \frac{7}{8}x + \frac{9}{8}$$