MAT 104 ASSIGNMENT

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Question

Examine whether or not there pair of lines are perpendicular to each other.

(1) $y-3x-2=0$ and $3y+x+9=0$

Solution.

For two lines to be perpendicular to each other, $M_1M_2=-1$

$y-3x-2=0$

$y=3x+2$

By Comparing with $y=mx+c$

$m_1=3$.

Also, $3y+x+9=0$

$3y=-x+9$

$y=\frac{-x}{3}+3$

$M_2=-1/3$

$M_1M_2=$ ~~AX~~ $3\times\frac{-1}{3}=-1$

Since $M_1M_2=-1$, the pair of lines $y-3x-2=0$ and $3y+x+9=0$ are said to be perpendicular to each other.

(2) $3y-4=2x+3$ and $y-5=x+6$.

Solution.

$3y=2x+3+4$

$3y=2x+7$

$y=\frac{2}{3}x+7/3$

$M_1=\frac{2}{3}$

$y-5=x+6$

$y=x+6+5$

$y=x+11$ $(y=(m)x+c$

$M_2=1$

For the two lines to be perpendicular to each other, $M_1M_2=-1$

$\frac{2}{3}\cdot 1=\frac{2}{3}\ (\neq -1)$

Since $M_1 M_2$ is not equal -1, The pair of lines are **not** perpendicular to each other.

3. $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution.

$$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y \cdot 1\right) - 0 = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2x + \frac{dy}{dx}(2y + 3x) + 3y = 0$$

$$\frac{dy}{dx}\frac{(2y+3x)}{2y+3x} = \frac{-2x - 3y}{2y + 3x}$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-2(1) - 3(2)}{2(2) - 3(1)} = \frac{-8}{7}$$

$$M_1 = \frac{-8}{7}$$

For Equation of tangent, $M_1 = M_2$

$$y - y_1 = M_1(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ → Equation of the tangent.

For Equation of normal $M_2 = \frac{-1}{M_1}$

$$y - y_1 = \frac{-1}{M_1}(x - x_1)$$

$$y - 2 = \frac{-1}{\frac{-8}{7}}(x-1)$$

$$y - 2 = \frac{7}{8}(x-1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0$$

$\therefore$ The Equation of the normal is $8y - 7x - 9 = 0$