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1. Function $y = \frac{1}{x} - 2$

Solution

The function is defined for all real numbers except $x = 2$

Domain = Real numbers except $x = 2$

Codomain = Real numbers except $y = 0$

2. If $k = \ln y$, differentiate k

$$\frac{d}{dk}(\ln y) = \frac{1}{y}$$

3. Express y as an explicit function of x if

a) $2x - 3y - 2 = 0$

$2x - 2 = 3y$

$$y = \frac{2x + 2}{3}$$

b) $x^2 + y^2 = 4$

$x^2 - 4 = y^2$

$y = \pm\sqrt{x^2 - 4}$

4) If $P = \sin^{-1} t$, find the derivative of P

$$P = \frac{t}{\sin}$$

$t = \sin P$ - - - ①

Recall that; $\sin^2 P + \cos^2 y = 1$ ②

$\frac{dt}{dp}$ of $(T) = \cos P$

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from (a) $\sin^2 P + \cos^2 P = 1$

$\cos^2 P = 1 - \sin^2 P$

$\cos P = \sqrt{1 - \sin P}$

$\cos P = \sqrt{1 - t^2}$

$\frac{dt}{dP} = \cos P = \sqrt{(1-t)^2}$

$\therefore \frac{dP}{dt} = \frac{1}{\sqrt{1-t^2}}$

5) $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $fog(x)$ and $gof(x)$

a) $(fog)x = f[g(x)]$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$f[g(x)] = f(4x - 2)$

$= 2(4x-2)^2 - 5$

$= (8x-4)^2 5$

$= 64x^2 - 32x - 32x + 16 - 5$

$= 64x^2 - 64 + 11$

b) $(gof)x$

$g[f(x)] = g(2x^2 - 5)$

$= 4\cancel{x} - 2[2x^2 - 5]$

$= 4(2x^2 - 5) - 2$

$= [8x^2 - 20] - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

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6) If $f(x) = 3x^2 - 2x + 1 = 0$, show that $f_e(x) + f_o(x = f(x)$.

$f_e(x) = \frac{f(x) + (-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$f(-x) = 3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2}$

$= \frac{6x^2 + 2}{2}$

$= \frac{\cancel{2}(3x^2 + 1)}{\cancel{2}}$

$\therefore f_e(x) = 3x^2 + 1$

$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$f_o(x) = \frac{3x^2 - 2x + 1 - 3x^2 - 2x}{2}$

$= -\frac{4x}{2} = -2x$

$\therefore f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

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7) $y = \cos x$ from 1st Principle

$y = \cos x$

$y = \delta y - \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x + \delta x) - y)$

but $y = \cos x$

$\therefore \delta y = \cos(x + \delta x) - \cos x$ - - - - 1*

consider from trig

$\cos(A + B) = \cos A \cos B \;\; \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ - - - - 2*

Compare (1*) and (2*)

let $A + B = x + \delta x$ - - - - (i)

$A - B = x$ - - - - (ii)

adding (i) and (ii)

$2A = 2x + \delta x$

$A = \dfrac{2x + \delta x}{2} \Rightarrow A = x + \dfrac{\delta x}{2}$ (3)

Substitute equation (3) in equation (2)

$x + \dfrac{\delta x}{2} - B = x$

$B = \dfrac{\delta x}{2}$

Compare equ(1*) and (2*)

$\cos(x + \delta x) - \cos x = -2\sin[x + {}^{\delta x}/_{2}] \sin[{}^{\delta x}/_{2}]$

$\therefore \delta y = -2\sin[x + {}^{\delta x}/_{2}] \sin[{}^{\delta x}/_{2}]$

$\dfrac{\delta y}{\delta x} = \dfrac{-2\sin[x + {}^{\delta x}/_{2}] \sin[{}^{\delta x}/_{2}]}{\delta x}$

$\dfrac{\delta y}{\delta x} = \dfrac{-\sin[x + {}^{\delta x}/_{2}] \sin[{}^{\delta x}/_{2}]}{{}^{\delta x}/_{2}}$

$\dfrac{\delta y}{\delta x} = -\sin[x + {}^{\delta x}/_{2}] \dfrac{\sin[{}^{\delta x}/_{2}]}{{}^{\delta x}/_{2}}$ - - - - (4)

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A standard limit

$$\lim_{\delta x \to} = \frac{\sin\left[\delta x/_2\right]}{\delta x/_2} = 1$$

Find limit of as $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim -\sin\left[x + \delta x/_2\right] \frac{\sin\left[\delta x/_2\right]}{\delta x/_2}$$

$$= -\sin\left[x + 0\right]$$

$$= -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{\delta y}{\delta x} = -\sin x //$$

8) Find $\delta y/\delta x$ if $y = 3t^2$ and $x = 1/t^2$

$y = \cos$ $\quad \delta x/\delta t = -2t \qquad \delta y/\delta t = 6t$

$$\delta y/\delta x = \frac{\delta y/\delta t}{\delta x/\delta t}$$

$$= \frac{6t}{-2t}$$

$$= -3t$$

9.) Find $\delta y/\delta x$ if $y = x^2 \cos\ 2x e^{4x}$

$y = x^2 \cos 2x e^{4x}$

$y = x^2 \cos\left(2xe^{4x}\right)$

using the product rule

$$\frac{\delta y}{\delta x} = V\frac{\delta u}{\delta x} + V\frac{\delta v}{\delta x}$$

let $u = x^2$ and $v = \cos 2xe^{4x}$

$$\frac{du}{dx} = 2x$$

$U = \cos\left(2xe^{4x}\right)$

using the chain rule $\delta y/\delta x = \delta u/\delta x \times \delta y/\delta y$

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$u = 2xe$ and $y = \cos u$

$\delta u/dx = 2 \times 4 \times e^{4x}$ and $y = -\sin u$

$= 8e^{4x}$ and $y = -\sin u$

$\delta u/\delta x = 8e^{4x} \times -\sin(2xe^{4x})$

$= -8e^{4x} \sin(2xe^{4x})$

$\delta y/\delta x = \cos(2xe^{4x}) \times 2x + x^2 \times 8e^{4x} \sin(2xe^{4x})$

$= 2x\cos(2xe^{4x}) + -8x^2e^{4x} \sin(2xe^{4x})$

10) Given that $y = \sin(3x^3+5)$ find the derivative of y

$y = \sin(3x^2+5)$

$\delta y/\delta x = \frac{\delta y}{\delta x} \times \frac{\delta u}{\delta y}$

$u = 3x^2+5 \quad y = \sin u$

$= 6x \quad y = \cos u$

$\delta y/\delta x = 6x \times \cos u$

$= 6x\cos u$

Since $u = 3x^2+5$

$\delta y/\delta x = 6x\cos(3x+5)$