MAT 104 MBBS 19/MHS01/424

Examine whether or not these pair of lines are perpendicular to each other

1 $y - 3x - 2 = 0$ —— ①

$3y + x + 9 = 0$ —— ②

soln

For two lines to be perpendicular, the product of their gradients must be equal

$\therefore m_1 m_2 = -1$

In line 1, $y - 3x - 2 = 0$

$\therefore y = 3x + 2$

By comparing with $y = mx + c$

$\therefore m_1 = 3$

In line 2, $3y + x + 9 = 0$

$\therefore 3y = -x - 9$

$y = -\frac{x}{3} - 9/3$

By comparing with $y = mx + c$

$\therefore m_2 = -\frac{1}{3}$

$\therefore m_1 m_2 = 3 \times -\frac{1}{3} = -1$

Since $m_1 m_2 = -1$, the lines are perpendicular to each other.

2 $3y - 4 = 2x + 3$ —— ①

$y - 5 = x + 6$ —— ②

soln

In line 1, $3y - 4 = 2x + 3$

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$\therefore y = 2x/3 + 7/3$

By comparing with $y = mx + c$

$m_1 = 2/3$

In line 2, $y-5 = x+6$

$$y = x+6+5$$

$$y = x+11$$

By comparing with $y = mx+c$

$$\therefore m_2 = 1$$

$$\therefore m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

Since $m_1 m_2 \neq -1$, the lines are not perpendicular to each other.

3 Find the equations of the tangent and normal to the curve

$$x^2 + y^2 + 3xy - 11 = 0 \text{ at point } x=1, y=2$$

$$x^2 + y^2 + 3(xy) - 11 = 0$$

$$\frac{dy}{dx} = m = 2x + 2y\frac{dy}{dx} + 3\left(y + x\frac{dy}{dx}\right) - 0 = 0$$

$$2x + 2y\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -3y - 2x$$

$$\frac{dy}{dx}(2y + 3x) = -3y - 2x$$

$$\therefore \frac{dy}{dx} = \frac{-3y-2x}{2y+3x}$$

$$\therefore m = \frac{-3y-2x}{2y+3x}$$

at $x=1, y=2$

$$\therefore m = \frac{-3(2)-2(1)}{2(2)+3(1)} = \frac{-6-2}{4+3} = \frac{-8}{7}$$

Equation of tangent $= (y-y_1) = m(x-x_1)$ where $m = \frac{-8}{7}$, $x=1, y=2$

$$\therefore y-2 = \frac{-8}{7}(x-1)$$

$$7(y-2) = -8(x-1)$$

$\frac{7}{8}$ $\frac{-8}{7}$

$7y - 14 = -8x + 8$

$\therefore 7y = -8x + 8 + 14$

$7y = -8x + 22$

Equation of normal ; $(y - y_1) = \frac{-1}{m}(x - x_1)$ where $m = -8/7$

$\therefore y - 2 = -(-7/8)(x - 1)$

$y - 2 = 7/8(x - 1)$

$8(y - 2) = 7(x - 1)$

$8y - 16 = 7x - 7$

$\therefore 8y = 7x - 7 + 16$

$8y = 7x + 9$