

ADEYI OREOLINDA OLUWATAYOBE

PHARMACOLOGY

MAT 104 (ASSIGNMENT)

19/MHS07/002

1 For what values of  $x$  is the function  $y = \sqrt{x-2}$  defined?

State the domain and codomain

→ The function is defined for all numbers except  $x=2$ .

→ The set of all real numbers except  $x=2$  (Domain) = The Co-domain is the set of all real number except  $y=0$ .

3 Express  $y$  as an explicit function of  $x$  in the following

a  $2x - 3y - 2 = 0$

b  $x^2 + y^2 = 4$

a  $2x - 3y - 2 = 0$

$$-3y = 2 - 2x$$

$$y = \frac{2 - 2x}{3}$$

$$\therefore y = \frac{2}{3} - \frac{2x}{3}$$

(b)  $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \sqrt{4 - x^2}$$

$$\therefore y = \sqrt{4 - x^2}$$

5 If  $f(x) = 2x^2 - 5$  and  $g(x) = 4x - 2$ , find  $f \circ g(x)$  and  $g \circ f(x)$

a  $f \circ g(x) = f(g(x))$

$$f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$f(g(x)) = f(4x - 2)$$

$$= 2(4x - 2)^2 - 5$$

$$= 2(4x - 2)(4x - 2) - 5$$

$$= 2(16x^2 - 8x - 8x + 4) - 5$$

$$= 32x^2 - 16x - 16x + 8 - 5$$

$$= 32x^2 - 32x + 3$$

$$= 32x^2 - 32x + 3$$

b  $g \circ f(x) = g(f(x))$

$$g(f(x)) = g(2x^2 - 5)$$

$$= 4(2x^2 - 5) - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22$$

6. If  $f(x) = 3x^2 - 2x + 1 = 0$ , show that  $f_0(x) + f_0(-x) = f(x)$ ,  
 $f_0(x) = \frac{f(x) + f(-x)}{2}$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$

$$f(-x) = 3x^2 + 2x + 1$$

$$f_0(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{6x^2 + 2}{2}$$

$$= 3x^2 + 1$$

$$f_0(x) = 3x^2 + 1$$

$$f_0(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$f_0(x) = \frac{3x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$f_0(x) = \frac{-4x}{2}$$

$$= -2x$$

$$\therefore f(x) = f_0(x) + f_0(x)$$

$$f(x) = 3x^2 + 1 - 2x$$

$$= 3x^2 - 2x + 1$$

$$\therefore f(x) = 3x^2 - 2x + 1$$

7. Differentiate  $y = \cos x$  from first principle

$$y = \cos x$$

$$y + \delta y = \cos(x + \delta x)$$

Subtract  $y$  from both sides

$$\delta y = \cos(x + \delta x) - y$$

$$\text{but } y = \cos x$$

$$\delta y = \cos(x + \delta x) - \cos x$$

Consider Trig

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$$

Let,

$$A + B = x + \delta x \quad \text{--- (1)}$$

$$A - B = x \quad \text{--- (2)}$$

$$2A = 2x + \delta x$$

$$A = \frac{2x + \delta x}{2}$$

$$A = x + \frac{\delta x}{2}$$

$$B = \frac{\delta x}{2}$$

$$\cos(x + \delta x) - \cos x = -2 \sin(x + \delta x/2) \sin(\delta x/2)$$

$$\delta y = 2 \sin(x + \delta x/2) \sin(\delta x/2)$$

$$\frac{\delta y}{\delta x} = \frac{2 \sin(x + \delta x/2) \sin(\delta x/2)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{\sin(x + \delta x/2) \sin(\delta x/2)}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = \sin(x + \delta x/2) \frac{\sin(\delta x/2)}{\delta x/2}$$

Standard limit

$$\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} -\sin(x + \delta x/2) \frac{\sin(\delta x/2)}{\delta x/2}$$

$$= -\sin(x + 0)$$

$$= -\sin x$$

$$\therefore \frac{dy}{dx} = -\sin x$$

9 Find  $\frac{dy}{dx}$  if  $y = x^2 \cos 2x e^{4x}$

$$y = x^2 \cos 2x e^{4x}$$

Find log of both sides

$$\ln y = \ln(x^2 \cos 2x e^{4x})$$

$$\ln y = \ln(x^2 + \cos 2x + e^{4x})$$

Differentiate both sides w.r.t  $x$

$$\frac{d}{dx} (\ln y) = \frac{d}{dx} (\ln x^2) + \frac{d}{dx} (\ln \cos 2x) + \frac{d}{dx} (\ln e^{4x})$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2x} \langle x^2 \rangle + \frac{1}{\sin 2x} \langle \cos 2x \rangle + \frac{1}{e^{4x}} \langle e^{4x} \rangle$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x^2}{2x} + \frac{\cos 2x}{\sin 2x} + \frac{e^{4x}}{e^{4x}}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{2} + \cot 2x + 1$$

Multiply both sides by  $y$

$$\frac{dy}{dx} = y \left\{ \frac{x}{2} + \cot 2x + 1 \right\}$$

$$\frac{dy}{dx} = x^2 \cos 2x e^{4x} \left\{ \frac{x}{2} + \cot 2x + 1 \right\}$$

$$\therefore \frac{dy}{dx} = x^2 \cos 2x e^{4x} \left\{ \frac{x}{2} + \cot 2x + 1 \right\}$$

10 Given that  $y = \sin \langle 3x^3 + 5 \rangle$  - Find the derivative of  $y$

$$y = \sin \langle 3x^3 + 5 \rangle$$

$$\text{Let } u = 3x^3 + 5 \quad \therefore \frac{du}{dx} = 9x^2$$

$$\Rightarrow y = \sin u \quad \therefore \frac{dy}{du} = \cos u$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \cos u \times 9x^2$$

$$\frac{dy}{dx} = 9x^2 \cos u$$

$$\text{but } u = 3x^3 + 5$$

$$\frac{dy}{dx} = 9x^2 \cos \langle 3x^3 + 5 \rangle$$

$$\therefore \text{the derivative of } y \text{ is } \frac{dy}{dx} = 9x^2 \cos \langle 3x^3 + 5 \rangle$$

2 If  $k = \ln V$ , differentiate  $k$ .

$$k = \ln V$$

Let  $k = y$  &  $V = x$

$$y = \ln x$$

Recall in logarithmic form

$$y = \ln x \Rightarrow y = \log_e x$$

$$x = e^y$$

So that,

$y = \ln(x)$  can be written as

$$e^y = x$$

$$\frac{d}{dx} \{e^y\} = e^y \frac{dy}{dx}$$

$$e^y \frac{dy}{dx} = x$$

When  $x$  is differentiated it gives 1

$$e^y \frac{dy}{dx} = 1$$

Recall  $e^y = x$

Sub

$$x \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{x}$$

8 Find  $\frac{dy}{dx}$  if  $y = 3t^2$  and  $x = \frac{1}{t^2}$

$$y = 3t^2 \quad ; \quad \frac{dy}{dt} = 6t$$

$$x = \frac{1}{t^2} \quad ; \quad \frac{dx}{dt} = \frac{-2t}{t^4}$$

Hence,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 6t \times \frac{t^4}{-2t}$$

$$= -3t^4$$

4 If  $p = \sin^{-1} t$ , find the derivative of  $p$

$$p = \sin^{-1} t$$

$$\text{Let } p = y \quad \& \quad t = x$$

$$\text{If } y = \sin^{-1} x = \arcsin(x)$$

$$y = \sin^{-1} x$$

$$y = \frac{x}{\sin}$$

$$x = \sin y$$

Differentiating both sides

$$\frac{dx}{dy} = \cos y$$

but we want  $\frac{dy}{dx}$ , therefore  $\frac{dy}{dx} = \frac{1}{\cos y}$

Recall,

$$\cos^2 y + \sin^2 y = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$\text{but } \sin y = x \Rightarrow \sin^2 y = x^2$$

$$\therefore \cos y = \sqrt{1 - x^2}$$

hence,

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{dy}{dx} \quad \text{but } y = p \quad \text{and } t = x$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos p} = \frac{1}{\sqrt{1 - t^2}}$$