

NAME: AIRANABEE EFFOOST ANTHONIA

DEPARTMENT: PHARMACY

MATRIC NUMBER: 19/MHS11/021

COURSE: MAT 102

Assignment

1. For what values of x is the function $y = \frac{1}{x+2}$ defined? State the domain and codomain?

Ans

The function is defined for all real numbers except $x = -2$.

The domain is all real numbers except $x = -2$

The codomain is all real numbers except $y = 0$

2. If $k = \ln v$ differentiate k

$$\frac{dk}{dv} = \frac{1}{v}$$

3. Express y as an explicit function of x

a $2x - 3y - 2 = 0$

$$\frac{-3y}{-3} = \frac{2 - 2x}{-3}$$

$$y = \frac{2x - 2}{3}$$

b $x^2 + y^2 = 4$

$$\sqrt{y^2} = \sqrt{4 - x^2}$$

$$y = \sqrt{4 - x^2}$$

4. If $P = \sin^{-1} t$ find derivative of P

$$P \times \frac{t}{\sin}$$

\sin

$$t = \sin p \quad \dots \dots$$

differentiate both side

$$\frac{dt}{dp} = \cos p$$

$$\frac{dp}{dt} = \frac{1}{\cos p}$$

Remember $\cos^2 p + \sin^2 p = 1$

$$\therefore \cos p = \sqrt{1 - \sin^2 p}$$

$$\frac{dp}{dt} = \frac{1}{\sqrt{1 - \sin^2 p}}$$

$$t = \sin p$$

$$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}} //$$

5 $f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$$\begin{aligned} f \circ g(x) &= 2(4x - 2)^2 - 5 \\ &= 2(16x^2 - 16x + 4) - 5 \\ &= 32x^2 - 32x + 8 - 5 \\ &= 32x^2 - 32x + 3 // \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= 4(2x^2 - 5) - 2 \\ &= 8x^2 - 20 - 2 \\ &= 8x^2 - 22 // \end{aligned}$$

6 $f(x) = 3x^2 - 2x + 1 = 0$

$$f_e(x) + f_o(x) = f(x)$$

$$f_o(x) = \frac{f(x) + f(-x)}{2}$$

$$\begin{aligned} f_o(x) &= \frac{3x^2 - 2x + 1 + 3(-x)^2 - 2(-x) + 1}{2} \\ &= \frac{3x^2 + 2x + 1}{2} \end{aligned}$$

$$f_o(0) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$= \frac{-4x}{2} = -2x$$

$$f(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$f(x) + f(x) = 3x^2 + 1 + (-2x)$$

$$= 3x^2 - 2x + 1$$

$$3x^2 - 2x + 1 = 3x^2 - 2x + 1 //$$

7 $y = \cos x$ from first principle

$$y + \delta y = \cos(x + \delta x)$$

$$y + \delta y - y = \cos(x + \delta x) - y$$

$$\delta y = \cos(x + \delta x) - \cos x \quad \text{--- eqn 1}$$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B \quad \text{--- eqn 2}$$

Compare equation 1 and 2

$$\text{let } (A+B) = x + \delta x$$

$$\text{let } (A-B) = x$$

$$2A = 2x + \delta x = A = \frac{2x + \delta x}{2} = x + \frac{\delta x}{2}$$

$$B = \frac{\delta x}{2}$$

$$\cos(x + \delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$$

$$\frac{dy}{dx} = \frac{-2}{\delta x} \times \sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$= -\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right) \div \frac{\delta x}{2}$$

$$= -\sin\left(x + \frac{\delta x}{2}\right)\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$\lim_{\delta x \rightarrow 0} = -\sin\left(x + \frac{\delta x}{2}\right) - 1$$

$$= -\sin(x+0) = -1$$

$$= -\sin x$$

8 $\frac{dy}{dx}$ if $y = 3t^2$ and $x = \frac{1}{2}t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \frac{dy}{dt} = 6t \quad \frac{dx}{dt} = 2t^{-3}$$

$$\frac{dy}{dx} = \frac{6t}{2t^{-3}} = 6t \div \frac{1}{2t^3}$$

$$= 6t \times 2t^3$$

$$= 12t^4 //$$

9 $\frac{dy}{dx}$ if $y = x^2 \cos 2x e^{4x}$

ln of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

$$\frac{d \ln y}{dx} = \frac{d \ln x^2}{dx} + \frac{d \ln \cos 2x}{dx} + \frac{d \ln e^{4x}}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} \times 2x + \frac{1}{\cos 2x} \times -2 \sin 2x + \frac{1}{e^{4x}} \times 4e^{4x}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2x}{x^2} + \frac{-2 \sin 2x}{\cos 2x} + 4$$

multiply both sides by y

$$\frac{dy}{dx} = y \left(\frac{2x^{-1}}{x^2} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$\frac{dy}{dx} = x^2 \cos 2x e^{4x} \left(\frac{2x^{-1}}{\cos 2x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right) //$$

10. $y = \sin(3x^3 + 5)$

$$y = \sin u$$

$$u = 3x^3 + 5$$

$$\frac{dy}{du} = \cos u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 9x^2 \times \cos u$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos(3x^3 + 5) //$$