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Matric no: 19/mths01/159
Course Code: MAT 104 (2nd Portal Submission).
Serial no: 069.

(A) Lines are said to be perpendicular when $m_1 m_2 = -1$.

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution:

First line: $y - 3x - 2 = 0$

w.r.t. (1) $\frac{dy}{dx} - 3 - 0 = 0$

(m_1) $\frac{dy}{dx} = 3$

Second line: $3y + x + 9 = 0$

$3\frac{dy}{dx} + 1 + 0 = 0$

$3\frac{dy}{dx} = -1$

$\therefore \frac{dy}{dx}\ (m_2) = \frac{-1}{3}$.

$m_1 \cdot m_2 = \cancel{3} \cdot \frac{-1}{\cancel{3}}$

$\therefore m_1 m_2 = -1$.

$\therefore$ These pair of lines are said to be perpendicular, because the product of their gradient is equal to -1.

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(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$.

Solution:

First line: $3y - 4 = 2x + 3$

$3y - 2x - 4 - 3 = 0$

$3y - 2x - 7 = 0$

$3\frac{dy}{dx} - 2 - 0 = 0$

$3\frac{dy}{dx} = 2$

$\therefore \frac{dy}{dx}(m_1) = \frac{2}{3}$.

Second line: $y - 5 = x + 6$

$y - x - 5 - 6 = 0$

$y - x - 11 = 0$

$\frac{dy}{dx} - 1 - 0 = 0$

$\frac{dy}{dx}(m_2) = 1$.

$m_1 \cdot m_2 = \frac{2}{3} \cdot 1$

$\therefore m_1 m_2 = \frac{2}{3}$.

$\therefore$ These pairs of lines are not perpendicular because the product of their gradient is not equal to -1.

(3.) $x^2 + y^2 + 3xy - 11 = 0$. $(x=1, y=2)$.

Solution

$$\left[2x + 2y\frac{dy}{dx} + 3\left(y + x\frac{dy}{dx}\right) - 0\right] = 0$$

$$2x + 2y\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(2y + 3x\right) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\text{OR} = -\frac{(2x + 3y)}{2y + 3x}$$

$$\therefore \frac{dy}{dx}(m) = -\frac{(2\cdot 1 + 3\cdot 2)}{2(2) + 3(1)}.$$

at $(x=1, y=2)$

$$= -\frac{(2+6)}{4+3}$$

$$= -\frac{8}{7}.$$

$\therefore \frac{dy}{dx}(m) = -\frac{8}{7}$

ⓐ Equation of the tangent:

$$(y - y_1) = m(x - x_1)$$

$$\frac{(y-2)}{1} = \frac{-8}{7}(x-1)$$

$$7(y-2) = -8(x-1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$\therefore$ The equation of the tangent is

$$7y + 8x - 22 = 0.$$

ⓑ Equation of the normal.

$$(y - y_1) = \frac{-1}{m}(x - x_1)$$

$$\frac{(y-2)}{1} = \frac{7}{8}(x-1)$$

$$8(y-2) = 7(x-1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$\therefore$ The equation of the normal is

$$8y - 7x - 9 = 0.$$