

3. Find the equations of the tangent and normal to the curve $x^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

$$2x + 3y \frac{dy}{dx} + 3(x + y \frac{dy}{dx}) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

When $x = 1$ and $y = 2$

$$m = - \frac{2[1] + 3[2]}{2(2) + 3(1)}$$

$$= - \frac{(2 + 6)}{4 + 3} = \frac{-8}{7}$$

$$m = \frac{-8}{7}$$

Equation of the tangent to a curve

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$y - 2 = \frac{-8x}{7} + \frac{8}{7}$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$8x + 7y - 22 = 0$$

b. Equation of the normal to a curve

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{-8/7}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$y - 2 = \frac{7x}{8} - \frac{7}{8}$$

$$8y - 16 = 7x - 7$$

Examine whether or not these pair of lines are perpendicular to each other.

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Let $D = y - 3x - 2 = 0$

$$= \frac{dy}{dx} - 3 = 0$$

$$\frac{dy}{dx} - 3 = 0$$

$$= \frac{dy}{dx} = 3$$

Let $A = 3y + x + 9 = 0$

$$3 \frac{dy}{dx} + 1 = 0$$

$$3 \frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -\frac{1}{3}$$

$A \perp B$

i.e. $y - 3x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Let $B = 3y - 4 = 2x + 3$

$$3 \frac{dy}{dx} - 0 = 2 + 0$$

$$3 \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

Let $C = y - 5 = x + 6$

$$\frac{dy}{dx} - 0 = 1 + 0$$

$$\frac{dy}{dx} = 1$$

$\therefore A \neq B$

i.e. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ is not perpendicular

$$s_y = 7x - 7 + 4$$

$$7x - s_y + 9 = 0$$