

MAT104 DR OYEELAMI

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MECHATRONICS ENGINEERING

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1. Find the integral of the following

a) $\int \frac{dx}{x^2+7}$

b) $\int \frac{dx}{x^2+64}$

Solution

a) $\int \frac{dx}{x^2+7}$

$$x = \sqrt{7} \tan \theta$$

$$\frac{dx}{d\theta} = \sqrt{7} \sec^2 \theta$$

$$dx = \sqrt{7} \sec^2 \theta d\theta$$

$$x^2 = 7 \tan^2 \theta$$

$$x^2+7 = 7(1+\tan^2 \theta)$$

$$x^2+7 = 7 \sec^2 \theta$$

where; $1+\tan^2 \theta = \sec^2 \theta$

$$\int \frac{\sqrt{7} \sec^2 \theta}{7 \sec^2 \theta} d\theta$$

$$\frac{\sqrt{7}}{7} \int d\theta$$

$$\frac{\sqrt{7}}{7} [\theta] + C$$

$$\theta = \tan^{-1} \frac{x}{\sqrt{7}}$$

$$\frac{\sqrt{7}}{7} \tan^{-1} \frac{x}{\sqrt{7}} + C$$

$$b. \int \frac{dx}{x^2+64} = \int \frac{dx}{x^2+8^2}$$

$$x = 8 \tan \theta$$

$$\theta = \tan^{-1} \frac{x}{8}$$

$$\frac{dx}{d\theta} = 8 \sec^2 \theta$$

$$dx = 8 \sec^2 \theta d\theta$$

$$x^2 = 64 \tan^2 \theta$$

$$x^2 + 64 = 64 (1 + \tan^2 \theta)$$

$$64 \sec^2 \theta$$

When $\theta = \tan^{-1} \frac{x}{8}$, $1 + \tan^2 \theta = \sec^2 \theta$

$$\int \frac{8 \sec^2 \theta}{64 \sec^2 \theta} d\theta$$

$$\frac{1}{8} \int d\theta$$

$$\frac{1}{8} [\theta] + C$$

$$\frac{1}{8} \tan^{-1} \frac{x}{8}$$