

19/EN905/017

1. A particle moves along a curve, $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$, where t is time. Find its velocity.

solution

$$\vec{r} = xi + yj + zk$$

$$\vec{r} = (7t^2)i + (6t^2 - 4t)j + (t - 5)k$$

$$\frac{d\vec{r}}{dt} = \text{velocity} = v$$

$$\frac{d\vec{r}}{dt} = (14t)i + (12t - 4)j + k$$

$$\text{velocity} = (14t)i + (12t - 4)j + k$$

2. If $A = i + 2j - 4k$, $B = 2i + 3j + k$, $C = 4j - 3k$, find $A \times (B \times C)$

solution

 $A \times (B \times C)$ $B \times C =$

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$B \times C = [(-3 \times 3) - 4]i - [(-3 \times 2) - 0]j + [(2 \times 4) - 0]k$$

$$5i + 6j + 8k$$

 $A \times (B \times C) =$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A \times (B \times C) = [16 - (-24)]i - [8 - (-20)]j + [6 - 10]k$$

$$40i - 28j - 4k$$

3 Given

$R = 4 \sin 3t i + 4e^{3t} j + 7t^3 k$, find the integral of R with respect to t

solution

$$\int R dt = \int \left[(4 \sin 3t) i + (4e^{3t}) j + (7t^3) k \right] dt$$

$$\int (4 \sin 3t) i dt + \int (4e^{3t}) j dt + \int (7t^3) k dt$$

$$\left(4 \times \frac{-1}{3} \cos 3t \right) i + \left(4 \times \frac{1}{3} e^{3t} \right) j + \left(\frac{7t^4}{4} \right) k$$

$$\int R dt = \left(\frac{-4}{3} \cos 3t \right) i + \left(\frac{4}{3} e^{3t} \right) j + \left(\frac{7t^4}{4} \right) k + C$$

4. If $A = 7i + 2j - k$, $B = 2i + j + 4k$, $C = i + j + k$, find $(A+C) \cdot (B-A)$

solution

$$\left[(A+C) \cdot (B-A) \right] = \left[(7i+2j-k) + (i+j+k) \right] \cdot \left[(2i+j+4k) - (7i+2j-k) \right]$$

$$\left[8i+3j+k \right] \cdot \left[-5i-j+5k \right]$$

$$(A+C) \cdot (B-A) = \left[8i+3j+k \right] \cdot \left[-5i-j+5k \right]$$

$$-40 - 3 + 0$$

$$(A+C) \cdot (B-A) = -43$$

5. Find a unit vector Tangent to the space curve $x=t$, $y=t^2$, $z=t^3$ at the point where $t=1$

solution

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{r} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$$

$$T = \frac{d\vec{r}/dt}{\left| \frac{d\vec{r}}{dt} \right|}$$

$$\frac{d\vec{r}}{dt} = i + 2tj + 3t^2k \quad \text{where } t = 1$$

$$\left. \frac{d\vec{r}}{dt} \right|_{t=1} = i + 2j + 3k$$

$$\left| \frac{d\vec{r}}{dt} \right|_{t=1} = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14} \approx 3.74$$

$$\text{Hence, } T = \frac{d\vec{r}}{dt} = \frac{i + 2j + 3t^2k}{3.74}$$

$$T = \frac{\left| \frac{d\vec{r}}{dt} \right|}{\left| \frac{d\vec{r}}{dt} \right|} = \frac{i + 2j + 3t^2k}{3.74}$$