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Q. Examine whether or not these pair of lines are perpendicular to each other

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

(i) Find the equations of the tangent and normal to the curve $x^2 + y^2 = 4$ at the point $x = 1, y = 2$

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

making y the subject of the formulae

$y = 3x + 2$ (by comparison with $y = mx + c$)

$m_1 = 3$

$3x + x + 9 = 0$

making y the subject of the formulae

$\frac{3y}{3} = \frac{-9-x}{3}$

$y = \frac{-x-9}{3} - 3$

$y = mx + c, m_2 = \frac{-1}{3}$

$m_1 m_2 = -1$

$3 \times \frac{-1}{3} = -1$ Since $m_1 m_2 = -1$, then the lines $y - 3x - 2 = 0$ and

$3y + x + 9 = 0$ are perpendicular.

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

let A be $3y - 4 = 2x + 3$, let B be $y - 5 = x + 6$

$3 \frac{dy}{dx} - 0 = 2 + 0$

$- 3 \frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{-3}$

B = $\frac{dy}{dx} = 1 + 0 = 1 + 0$

$\frac{dy}{dx} = 1$

$A \neq B$

then $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular.

$$3. x^2 + y^2 + 3xy - 11 = 0$$

$\frac{dy}{dx}$

$$2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \cdot 1 \right) = 0$$

$$2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \right) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} / x=1, y=2 = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = 8(x - 1) = 7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$8x + 7y - 22 = 0$$

Equation of the normal of a curve

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 2 = \frac{-1}{-8/7}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

Collect like terms

$$8y = 7x - 7 + 16$$

$$7x - 8y - 7 + 16 = 0$$

$$7x - 8y + 9 = 0$$