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191MHS 01/300

Medicine and Surgery

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1. Examine whether or not these pairs of lines are perpendicular to each other:

a)  $y - 3x - 2 = 0$  and  $3y + x + 9 = 0$

Let  $y - 3x - 2 = 0$  be  $m_1$

and  $3y + x + 9 = 0$  be  $m_2$

$\therefore$  for  $m_1$

$$\Rightarrow y - 3x - 2 = 0 \Rightarrow y = 3x + 2$$

$$\Rightarrow \frac{dy}{dx} = 3 + 0$$

$$dx$$

$$\Rightarrow \frac{dy}{dx} = 3$$

$$dx$$

$\therefore$  for  $m_2$

$$\Rightarrow 3y + x + 9 = 0 \Rightarrow 3y = -x - 9$$

$$\Rightarrow 3 \frac{dy}{dx} = -1 - 0$$

$$dx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{3}$$

$$dx$$

$$\therefore m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

$$\therefore \Rightarrow m_1 m_2 = -1$$

$$3 \times -\frac{1}{3} = -1$$

$\therefore y - 3x - 2 = 0$  is perpendicular to  $3y + x + 9 = 0$

2 b)  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$

Let:  $3y - 4 = 2x + 3$  be  $m_1$

and  $y - 5 = x + 6$  be  $m_2$

$\therefore$  for  $m_1$

$$3y - 4 = 2x + 3$$

$$\Rightarrow 3y = 2x + 3 + 4 \Rightarrow 3 \frac{dy}{dx} = 2$$

$$dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{3}$$

∴ for  $m_2$

$$y - 5 = x + 6$$

$$\Rightarrow y = x + 6 + 5$$

$$\Rightarrow \frac{dy}{dx} = 1$$

$m_2$

$$\therefore m_1 = \frac{2}{3} \text{ and } m_2 = 1$$

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$$\Rightarrow \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\therefore m_1, m_2 \neq -1$$

∴  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$  are not perpendicular to each other

3 Find the equations of the tangent and normal to the curve;  $x^2 + y^2 + 3xy - 11 = 0$  at the point  $x = 1, y = 2$

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} + 3 \left( x \frac{dy}{dx} + y \right) - 0 = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\Rightarrow \frac{dy}{dx} (2y + 3x) = -3y - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3y - 2x}{2y + 3x}$$

$$m = \frac{dy}{dx}$$

$$\frac{dy}{dx} \Big|_{x=1 \text{ and } y=2} \Rightarrow \frac{-3(2) - 2(1)}{2(2) + 3(1)} \Rightarrow \frac{-6 - 2}{4 + 3} \Rightarrow \frac{-8}{7}$$

$$\Rightarrow m = \frac{-8}{7}$$

Equation for tangent =  $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = \frac{-8}{7}(x - 1)$$

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$$\Rightarrow 7(y-2) = -8(x-1)$$

$$\Rightarrow 7y - 14 = -8x + 8$$

$$\Rightarrow 8x + 7y - 14 + 8 = 0$$

$$\Rightarrow 8x + 7y - 14 - 8 = 0$$

$$\Rightarrow 8x + 7y - 22 = 0$$

Equation for normal:  $m_1 m_2 = -1$

$$\Rightarrow m_2 = \frac{-1}{m_1} \Rightarrow \frac{-1}{-8/7} \Rightarrow +1 \times \frac{7}{8} \Rightarrow \frac{7}{8}$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 16 = 7x - 7$$

$$\Rightarrow 7x - 8y + 16 - 7 = 0$$

$$\Rightarrow 7x - 8y + 9 = 0$$

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