

MATHS 104

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MBBS

Question: Examine whether or not these pair of lines are perpendicular to each other.

1)  $y - 3x - 2 = 0$  and  $3y + x + 9 = 0$

2)  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$

3) find the equations of the tangent and normal to the curve  $x^2 + y^2 + 3xy - 11 = 0$  at the point  $x=1, y=2$

Solution

1)  $y - 3x - 2 = 0$  and  $3y + x + 9 = 0$

let  $y - 3x - 2 = 0$  ;  $M_1$

$3y + x + 9 = 0$  ;  $M_2$

$\therefore M_1$

$\Rightarrow y - 3x - 2 = 0 \rightarrow y = 3x + 2$

$\Rightarrow \frac{dy}{dx} = 3 + 0$

$\frac{dy}{dx} = 3$

$\therefore M_2$

$3y + x + 9 = 0$

$3y = -x - 9$

$3 \frac{dy}{dx} = -1 - 0$

$\frac{dy}{dx} = -\frac{1}{3}$

$\therefore M_1 = 3 ; M_2 = -\frac{1}{3}$

$M_1 M_2 = -1$

$3 \times -\frac{1}{3} = -1$

$y - 3x - 2 = 0$  is perpendicular to  $3y + x + 9 = 0$

$$2) 3y - 4 = 2x + 3 \text{ --- (1)}$$

$$y - 5 = x + 6 \text{ --- (2)}$$

Making  $y$  the subject of formulae from eq. (1)

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

Using  $y = mx + c$

$$m_1 = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 6 + 5$$

$$y = x + 11$$

Using  $y = mx + c$

$$m_2 = 1$$

$$\therefore m_1 m_2 = -1$$

$$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

Hence the lines  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$  are not perpendicular.

$$3) x^2 + y^2 + 3xy - 11 = 0 \quad (x=1, y=2)$$

$$m = \frac{dy}{dx}$$

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx}$$

$$2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} \Big|_{x=1, y=2} = \frac{2(1) - 3(2)}{2(2) + 3(1)} = \frac{2 - 6}{4 + 3} = \frac{-4}{7} = -\frac{4}{7}$$

a) Equation of a tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{4}{7}(x - 1)$$

$$7(y - 2) = -4(x - 1)$$

$$7y - 14 = -4x + 4$$

$$7y + 4x - 14 - 4 = 0$$

$$7y + 4x - 18 = 0$$

b) Equation of normal

$$y - y_1 = m(x - x_1)$$

$$m_1 m_2 = -1$$

$$m_2 = \frac{-1}{m_1}$$

$$= \frac{-1}{-\frac{4}{7}} = \frac{7}{4}$$

$$y - 2 = \frac{7}{4}(x - 1)$$

$$4(y - 2) = 7(x - 1)$$

$$4y - 8 = 7x - 7$$

$$4y = 8 - 7x + 7 = 0$$

$$4y = 7x - 8 + 7 = 0$$

$$4y - 7x + 9 = 0$$