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MAT 104

Assignment

Examine whether or not these pairs of lines are perpendicular to each other.

- 1) $y - 3x - 2 = 0$ & $3y + x + 9 = 0$
- 2) $3y - 4 = 2x + 3$ & $y - 5 = x + 6$

3) Find the equations of the tangent & normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solutions:-

1) $y - 3x - 2 = 0$ & $3y + x + 9 = 0$

They are to be perpendicular if $m_1 m_2 = -1$

$$y - 3x - 2 = 0$$

$$y = 3x + 2$$

$$y = mx + c$$

$$\therefore m_1 = 3$$

$$3y + x + 9 = 0$$

$$3y = -x - 9$$

$$y = \frac{-x}{3} - \frac{9}{3}$$

$$y = \frac{-1x}{3} - 3$$

$$y = mx + c$$

$$\therefore m_2 = -\frac{1}{3}$$

$$m_1 m_2 = -1, \quad 3 \times -\frac{1}{3} = -1$$

\therefore They are perpendicular to each other

2) $3y - 4 = 2x + 3$ & $y - 5 = x + 6$

They are to be perpendicular if $m_1 m_2 = -1$

$$3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x}{3} + \frac{7}{3}$$

$$y = mx + c$$

$$\therefore m_1 = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 6 + 5$$

$$y = x + 11$$

$$y = mx + c$$

$$y = 1$$

$$m_2 = 1$$

$$m_1 m_2 = -1, \quad \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\therefore m_1 m_2 \neq -1$$

They are not perpendicular to each other

3) ~~Find the~~ $x^2 + y^2 + 3xy - 11 = 0$ at the point $(1, 2)$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y) = 0$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$2y + 3x$$

$$\frac{dy}{dx} \Big|_{x=1, y=2}$$

$$\frac{dy}{dx} \Big|_{x=1, y=2}$$

$$= \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

$$m = -8/7$$

Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$$7y + 8x - 22 = 0$$

Equation of the normal

$$y - y_1 = \frac{1}{m}(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0$$