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19/ENG 06/052

Mechanical Engineering

$$(ii) A \times (B \times C)$$

$$B \times C = \begin{vmatrix} i & j & k \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$= i \begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - j \begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + k \begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$$

$$= i(-5+44) - j(-15+44) + k(12-4)$$

$$(B \times C) = 39i - 29j + 8k$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 39 & 29 \end{vmatrix}$$

$$= i(-8+0) - j(16-0) + k(58+39)$$

$$= -8i - 16j + 97k$$

$$(iv) (3A \times B) \cdot (A \times 2B)$$

$$3A = 3(2i - j)$$

$$= 6i - 3j$$

$$3A \times B = \begin{vmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix} = i \begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - j \begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + k \begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= i(33-0) - j(-66-0) + k(6+9)$$

$$= 33i + 66j + 15k$$

$$2B = 2(3i + j - 11k)$$

$$= 6i + 2j - 22k$$

$$A \times 2B = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$= i(22 - 0) - j(-44 - 0) + k(4 + 6)$$

$$= 22i + 44j + 10k$$

$$\therefore (3A \times B) \cdot (A \times 2B)$$

$$= (33i + 66j + 5k) \cdot (22i + 44j + 10k)$$

$$= 726 + 2904 + 50$$

$$= 3680 //$$

$$(v) A - 2B - C$$

$$= 2i - j - 2(3i + j - 11k) - (4i + 4j - 5k)$$

$$= 2i - j - 6i - 2j + 22k - 4i - 4j + 5k$$

$$= 2i - 6i - 4i - j - 2j - 4j + 22k + 5k$$

$$= -8i - 7j + 27k$$

2) - Perpendicularity of vectors
Two vectors A & B are said to be perpendicular if $\vec{A} \cdot \vec{B} = 0$

Co-planarity of vectors

Three vectors A, B & C are said to be co-planar if $A \cdot (B \times C) = 0$

$$A = 2i - j$$

$$B = 3i + j - 11k$$

$$C = 4i + 4j - 5k$$

$$(i) -3A + 7B - 8C$$

$$= -3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k)$$

$$= -6i + 3j + 21i + 7j - 77k - 32i - 32j + 40k$$

$$= -6i + 21i - 32i + 3j + 7j - 32j - 77k + 40k$$

$$= -17i - 22j - 37k$$

$$(ii) K = 2A + 4B - C$$

$$K = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$$

$$= 4i - 2j + 12i + 4j - 44k - 4i - 4j + 5k$$

$$= 4i + 12i - 4i - 2j + 4j - 4j - 44k + 5k$$

$$= 12i - 2j - 39k$$

$$|K| = \sqrt{12^2 + (-2)^2 + (-39)^2}$$

$$= \sqrt{144 + 4 + 1521}$$

$$= \sqrt{1669}$$

$$= 40.85$$

Direction cosine \Rightarrow

$$L = \cos \alpha = \frac{12}{40.85}$$

$$M = \cos \beta = \frac{-2}{40.85}$$

$$N = \cos \gamma = \frac{-39}{40.85}$$