

ELSMARI ZAHAB DAMILOLA

M/111501/426

MEDICINE AND SURGERY

MAT 104

Examine whether or not these pair of lines are perpendicular to each other:

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Two lines with gradients m_1 and m_2 are said to be perpendicular if the product of their gradients equals -1

i.e. $m_1 m_2 = -1$

$y - 3x - 2 = 0$

gradient $m_1 = \frac{dy}{dx} =$

$m_1 \frac{dy}{dx} - 3 = 0$

$\frac{dy}{dx}$

$\frac{dy}{dx} = 3$

$\frac{dy}{dx}$

$3y + x + 9 = 0$

gradient $m_2 = \frac{dy}{dx} =$

$m_2 : 3 \frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx}$

$3 \frac{dy}{dx} = -1$

$\frac{dy}{dx}$

$\frac{dy}{dx} = -1/3$

$\therefore m_1 = 3$ and $m_2 = -1/3$

for perpendicular lines: $m_1 m_2 = -1$

$3 \times -1/3 = -1$

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$\therefore y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular to each other because the product of their gradient $= -1$

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

$3y - 4 = 2x + 3$

$3 \frac{dy}{dx} - 0 = 2 + 0$

$\frac{dy}{dx}$

$3 \frac{dy}{dx} = 2$

$m_1 = \frac{dy}{dx} = 2/3$

$\therefore m_1 = 2/3$ and $m_2 = 1$

for perpendicular lines: $m_1 m_2 = -1$

$2/3 \times 1 = 2/3$

(SM) ZAIRAB DANILOLA
19/11/2019/426
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$$m_1 m_2 \neq -1$$

$3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular to each other because the product of their gradients is not equal to -1 .

3. Find the equations of the tangent and normal to the curve

$$x^2 + y^2 + 3xy - 11 = 0 \text{ at the point } x = 1, y = 2$$

$$x_1 = 1, y_1 = 2$$

$$x^2 + y^2 + 3xy - 11 = 0$$

Gradient $m_1 = \frac{dy}{dx}$:

$$2x + 2\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y \cdot 1\right) = 0 \Rightarrow 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 2x + 3y = 0$$

$$\frac{dy}{dx}(2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\frac{dy}{dx} \text{ at } x=1 \text{ and } y=2 = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$$

$$\therefore m_1 = -8/7$$

Equation of tangent:

$$y - y_1 = m_1(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$$7y + 8x - 22 = 0 \text{ (which gives the equation of the tangent)}$$

$$m_1 m_2 = -1$$

$$\frac{-8}{7} m_2 = -1$$

$$m_2 = -1 \div \frac{-8}{7} = +1 \times \frac{7}{8} = \frac{7}{8} \therefore m_2 = 7/8$$

USMAN LATIF DAMILOLA
19/11/201/426
MBBS

~~Ans~~ → Equation of the normal:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0 \quad (\text{which gives the equation of the normal})$$