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19/ENG05/048 ⇒ MECHATRONICS ENGR.
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Find the

(1) $\int \frac{dx}{x^2+7}$

(2) $\int \frac{dx}{x^2+64}$

Solution

(1) Let $x = \sqrt{7}\tan\theta$ from $x = a\tan\theta$

$\frac{dx}{d\theta} = \sqrt{7}\sec^2\theta$

$dx = \sqrt{7}\sec^2\theta\, d\theta$

$7+x^2 = 7+(\sqrt{7}\sec^2\theta)^2$

$7+x^2 = 7+(\sqrt{7}\tan\theta)^2$

$7+x^2 = 7+7\tan^2\theta$

$= 7(1+\tan^2\theta)$

Recall that $1+\tan^2\theta = \sec^2\theta$

$\therefore 7+x^2 = 7\sec^2\theta$

$\therefore \int \frac{dx}{x^2+7} = \int \frac{\sqrt{7}\sec^2\theta\, d\theta}{7\sec^2\theta}$

$= \frac{\sqrt{7}}{7}\int d\theta$

$= \frac{\sqrt{7}}{7}[\theta] + c$

And $\theta = \tan^{-1} x/\sqrt{7}$

$\therefore \int \frac{dx}{x^2+7} = \frac{\sqrt{7}}{7}\tan^{-1} x/\sqrt{7} + c$ ✓

(2) $\int \frac{dx}{x^2+64}$

Solution:

$\int \frac{dx}{x^2+64} = \int \frac{dx}{x^2+8^2}$

Since $x = a\tan\theta$

from $\int \frac{dx}{x^2+a^2}$ $\therefore a = 8$

$\therefore$ let $x = 8\tan\theta$

$\frac{dx}{d\theta} = 8\sec^2\theta$

$\therefore dx = 8\sec^2\theta\, d\theta$

$\therefore 64+x^2 = 64+(8\tan\theta)^2$

$= 8^2 + 8^2\tan^2\theta$

$= 8^2(1+\tan^2\theta)$

Recall that $1+\tan^2\theta = \sec^2\theta$

$\therefore 64+x^2 = 64\sec^2\theta$

$\therefore \int \frac{dx}{x^2+64} = \int \frac{8\sec^2\theta\, d\theta}{64\sec^2\theta}$

$= \frac{1}{8}\int d\theta$

$= \frac{1}{8}[\theta] + c$

Recall that $\theta = \tan^{-1} x/8$

$\therefore \int \frac{dx}{x^2+64} = \frac{1}{8}\tan^{-1} x/8 + c$ ✓