

**Image**

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COURSE: MAT 104

### Assignment

1) Examine whether or not these pair of lines are perpendicular to each other.

i)  $y - 3x - 2 = 0$  and  $3y + x + 9 = 0$

ii)  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$

Soln

2) i)  $y - 3x - 2 = 0$

$$y = 3x + 2$$

$$y = mx + c$$

$$\therefore m_1 = 3$$

$$3y + x + 9 = 0 \Rightarrow (x + 3y) = -9$$

$$3y = -x - 9$$

$$y = \frac{-x - 9}{3} = \frac{-1}{3}x - 3$$

$$y = mx + c$$

$$\therefore m_2 = -\frac{1}{3}$$

lines are perpendicular if  $m_1 m_2 = -1$

$$3 \times -\frac{1}{3} = -1$$

$\therefore$  They are perpendicular.

ii

$$3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x + 7}{3}$$

$$y = mx + c$$

$$\therefore m_1 = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 6 + 5$$

$$y = x + 11$$

$$y = mx + c$$

$$\therefore m_2 = 1$$

$$1 = m_1 m_2 = -1$$

$\frac{2}{3} \times 1 = \frac{2}{3} \neq -1$   $\therefore$  They are not perpendicular.

2) Find the equations of the tangent and normal to the curve  $x^2 + y^2 + 3xy - 11 = 0$  at the point  $x=1, y=2$ .

soln  
 $x^2 + y^2 + 3xy - 11 = 0$

$$\frac{dy}{dx} = 2x + 3\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} - 0 = 0$$

$$\frac{dy}{dx} = 2x + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = 2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{dy}{dx}(2y+3x) = -2x-3y$$

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$\left. \frac{dy}{dx} \right|_{x=1, y=2} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$$

$$\frac{dy}{dx} = \frac{-2-6}{4+3} = \frac{-8}{7}$$

$$\frac{dy}{dx} = \frac{-8}{7}$$

$\therefore m_1 = -\frac{8}{7}, y_1 = 2, x_1 = 1$

Equation of a tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x = 22$$

$$7y + 8x = 22$$

$$7y + 8x - 22 = 0$$

$$7y + 8x - 22 = 0$$

Equation of the normal

$$m_1 m_2 = -1, \quad \frac{-8}{7} \times m_2 = -1$$

$$m_2 = -1 \times \frac{7}{8}$$

$$m_2 = -1 \times \frac{7}{8}$$

$$m_2 = \frac{7}{8}$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x = -7 + 16$$

$$8y - 7x = 9$$

$$8y - 7x - 9 = 0$$