6TH APRIL, 2020

AKPOFURE TESE

100 LEVEL

19/MHS01/077

MEDICINE AND SURGERY

MEDICINE AND HEALTH SCIENCES

MAT 104 - GENERAL MATHEMATICS III

ASSIGNMENT

Examine whether or not these pair of lines are perpendicular to each other:

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

Let A be $y - 3x - 2 = 0$

This can also be $y = 3x + 2$

Gradient, $m_1 = {}^{dy}/_{dx} = 3$

Let B be $3y + x + 9 = 0$

This can also be: $3y = -x - 9 \Rightarrow y = -{}^{1}/_{3}x - 3$

Gradient, $m_2 = {}^{dy}/_{dx} = -{}^{1}/_{3}$

For perpendicular lines, $m_1 \times m_2 = -1$

$3 \times -{}^{1}/_{3} = -1$

$\therefore$ $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular lines

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

Let A be $3y - 4 = 2x + 3$

This can be written as $3y = 2x + 7 \rightarrow y = {}^{2}/_{3}x + {}^{7}/_{3}$

Gradient, $m_1 = {}^{dy}/_{dx} = {}^{2}/_{3}$

Let B be $y - 5 = x + 6$

This can also be $y = x + 11$

Gradient, $m_2 = {}^{dy}/_{dx} = 1$

For perpendicular lines, $m_1 \times m_2 = -1$

$$\frac{2}{3} \times 1 = \frac{2}{3}$$

$$\frac{2}{3} \times \frac{1}{} \neq -1$$

$\therefore$ $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular.

3. Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

Solution

$$x^2 + y^2 + 3xy - 11 = 0$$

$$2x + 2y\frac{dy}{dx} + 3\left(x \times \frac{dy}{dx} + y \times 1\right) - 0 = 0$$

$$2x + 2y\,{}^{dy}/_{dx} + 3x\,{}^{dy}/_{dx} + 3y = 0$$

$$2y\,{}^{dy}/_{dx} + 3x\,{}^{dy}/_{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x} = \frac{(-2 \times 1) - (3 \times 2)}{(2 \times 2) + (3 \times 1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

$\therefore$ $m = {}^{-8}/_{7}$

Equation of tangent to a curve: $y - y_1 = m(x - x_1)$

At point $(1, 2)$: $y - 2 = {}^{-8}/_{7}(x - 1)$

$$y - 2 = -{}^{8}/_{7}x + {}^{8}/_{7}$$

$$y - 2 + {}^{8}/_{7}x - {}^{8}/_{7} = 0$$

$$7y - 14 + 8x - 8 = 0$$

$7y = 22 - 8x$ or $7y + 8x - 22 = 0$ gives the eqn of tangent.

Equation of normal to a curve: $y - y_1 = {}^{-1}/_{m}(x - x_1)$

At point $(1, 2)$: $y - 2 = -\left(\frac{-7}{8}\right)(x - 1)$

$$y - 2 = {}^{7}/_{8}x - {}^{7}/_{8}$$

$$y - 2 - {}^{7}/_{8}x + {}^{7}/_{8} = 0$$

$$8y - 16 - 7x + 7 = 0$$

$8y = 7x + 9$ or $8y - 7x - 9 = 0$ gives the equation of normal.