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MECHANICAL ENGINEERING
19/ENR06/051

MATH 102

if $A = 2i - j$, $B = 3i + j - 11k$ and $C = 6i + 6j - 5k$
find

i) $-3A + 7B - 8C$

$$-3(2i - j) + 7[3i + j - 11k] - 8[6i + 6j - 5k]$$

$$= 6i + 3j + 21i + 7j - 77k - 32j - 32j + 40k$$

$$[-6i + 21i - 32i] + [3j + 7j - 32j] + [-77k + 40k]$$

$$= -17i - 22j - 37k$$

ii) $R = 2A + 4B - C$ find direction cosine

$$R = 2[2i - j] + 4[3i + j - 11k] - 1[6i + 6j - 5k]$$

$$= 4i - 2j + 12i + 4j - 6k - 6k - 4j + 5k$$

$$= 16i + 2j - 7k$$

$$|R| = \sqrt{(16)^2 + (2)^2 + (-7)^2} = \sqrt{258} = 16.06$$

$$|R| = \sqrt{144 + 36 + 49} = 16.06$$

$$= \sqrt{258} = 16.06$$

$$x = \cos \alpha = 16/16.06$$

$$\alpha = \cos^{-1} \left[\frac{16}{16.06} \right]$$

$$\alpha = \cos^{-1} [0.9962]$$

$$\alpha = 1.53^\circ$$

$$y = \cos \beta = 2/16.06$$

$$\beta = \cos^{-1} \left[\frac{2}{16.06} \right]$$

$$\beta = \cos^{-1} [0.1245]$$

$$\beta = 1.105^\circ$$

$$z = \cos \gamma = -7/16.06$$

$$\gamma = \cos^{-1} \left[\frac{-7}{16.06} \right]$$

$$\gamma = \cos^{-1} [0.4358]$$

$$\gamma = 1.105^\circ$$

iii) $A \times [B \times C]$

$$[B \times C] = \begin{vmatrix} 1 & 6 & 10 \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & -11 & 0 \\ 4 & 5 & -15 \end{vmatrix} + 11 \begin{vmatrix} 3 & -11 \\ 4 & 5 \end{vmatrix} + 15 \begin{vmatrix} 3 & -11 \\ 3 & 11 \end{vmatrix}$$

$$= 1[-5+44] - 1[-15+44] + 15[12-4]$$

$$= 39i + 29j + 81k$$

$$A \times [B \times C] = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 39 & 29 & 8 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 39 & -29 \end{vmatrix}$$

$$= 8i - 16j + 97k$$

2. $A \times [B \times C] = 8i - 16j + 97k$

iv) $[3A \times B] \cdot [A \times 2B]$

$$3A = 3[2i - j] = 6i - 3j$$

$$= 6i - 3j$$

$$2B = 2[3i + j - 11k] = 6i + 2j - 22k$$

$$= 6i + 2j - 22k$$

$$3A \times B = \begin{vmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - 1 \begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + 11 \begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= 66i + 66j + 15k$$

$$A \times 2B = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + 11 \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$= 22i + 44j + 60k$$

$$[3A \times B] \cdot [A \times 2B]$$

$$[66i + 66j + 15k] \cdot [22i + 44j + 60k]$$

$$= 1452i + 2904j + 150k$$

$$6] A - 2B - C$$

$$2\hat{i} - \hat{j} - 6\hat{k} - 2(\hat{i} + 2\hat{k}) - 4\hat{i} + 6\hat{j} = 5\hat{k}$$
$$= -8\hat{i} + \hat{j} + 5\hat{k}$$

4] Vectors are said to be perpendicular if and only if their scalar product is equal to zero

5] Vectors are said to be coplanar if their scalar triple product is zero