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Course: Math 104

Dept: Medicine & Surgery

1. Examine where or not these pairs of lines are perpendicular to each other:

2. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

let $y - 3x - 2 = 0$ be m_1

and $3y + x + 9 = 0$ be m_2

\therefore for m_1

$$y - 3x - 2 = 0 \rightarrow y = 3x + 2$$

$$\frac{dy}{dx} = 3 + 0$$

$$\frac{dy}{dx} = 3$$

For m_2

$$3y + x + 9 = 0 \rightarrow 3y = -x - 9$$

$$\rightarrow 3 \frac{dy}{dx} = -1 - 0$$

$$\frac{dy}{dx} = \frac{-1}{3}$$

$$\therefore m_1 = 3 \text{ and } m_2 = -1/3$$

$$\therefore m_1 m_2 = -1$$

$$3 \times -1/3 = -1$$

$\therefore y - 3x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

let: $3y - 4 = 2x + 3$ be m_1

and $y - 5 = x + 6$ be m_2

\therefore for m_1

$$3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4$$

$$3 \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

for m_2

$$y-5=x+6$$

$$\rightarrow y=2+6+5$$

$$\frac{dy}{dx} = 1$$

$$\therefore m_1 = \frac{2}{3} \text{ and } m_2 = 1$$

$$\rightarrow \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\therefore m_1 m_2 \neq -1$$

$\therefore 3y-4=2x+3$ and $y-5=x+6$ are not perpendicular to each other.

3. Find the equations of the tangent and of normal to the curve:

$$x^2 + y^2 + 3xy - 11 = 0 \text{ at the point } (x=1, y=2)$$

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\rightarrow 2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \right) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\rightarrow \frac{dy}{dx} (2y + 3x) = -3y - 2x$$

$$\rightarrow \frac{dy}{dx} = \frac{-3y - 2x}{2y + 3x}$$

$$m = \frac{dy}{dx} \Big|_{x=1 \text{ and } y=2} \rightarrow \frac{-3(2) - 2(1)}{2(2) + 3(1)} \rightarrow \frac{-6 - 2}{4 + 3} = \frac{-8}{7}$$

$$\text{Equation of tangent} = y - y_1 = m(x - x_1)$$

$$\rightarrow y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 + (-8) = 0$$

$$8x + 7y - 14 - 8 = 0$$

$$8x + 7y - 22 = 0$$

Equation for normal: $m_1 m_2 = -1$

$$\rightarrow m_2 = \frac{-1}{m_1} \rightarrow \frac{-1}{-8/7} \rightarrow +1 \times \frac{7}{8} = \frac{7}{8}$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 16 = 7x - 7$$

$$\rightarrow 7x - 8y + 16 - 7 = 0$$

$$\rightarrow 7x - 8y + 9 = 0$$