OKORIE GLORY OLUWASEFUNMI
MEDICINE AND SURGERY
19/MHS01/323
MAT 104 ASSIGNMENT

1) Examine whether or not these pair of lines are perpendicular to each other.
i) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$
ii) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution.

i) $y - 3x - 2 = 0$ ——— (i)
$3y + x + 9 = 0$ ——— (ii)

From equ (i)
$y - 3x - 2 = 0$
Recall that: $y = mx + c$
$y = 3x + 2$
Hence from this; $m_1 = 3$

From equ (ii)
$3y + x + 9 = 0$
Recall that: $y = mx + c$
$3y = -x - 9$
$y = \frac{-x - 9}{3}$
$y = \frac{-x}{3} - \frac{9}{3}$

Here, coefficient of $x = \frac{-1}{3}$ which is m_2

Also, Recall that condition for perpendicularity $m_1 m_2 = -1$
$m_1 = 3$ $m_2 = \frac{-1}{3}$
$3x + \frac{1}{3} = -1$

Hence: $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular to each other.

ii) $3y - 4 = 2x + 3$ ——— (i)

$y - 5 = x + 6$ ——— (ii)

From equ (i)

$3y - 4 = 2x + 3$

Recall that $y = mx + c$

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2x + 7}{3}$

$y = \frac{2x}{3} + \frac{7}{3}$

Here, $m_1 = \frac{2}{3}$

From equ (ii)

$y - 5 = x + 6$

Recall that $y = mx + c$

$y = x + 6 + 5$

$y = x + 11$

Here, $m_2 = 1$

Also, Recall that condition for perpendicularity $\Rightarrow m_1 m_2 = -1$

$m_1 = \frac{2}{3}$ $m_2 = 1$

$\frac{2}{3} \times 1 = \frac{2}{3}$

Hence; $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular to each other.

2) Find the equations of the tangent and normal to the curve $x^2+y^2+3xy-11=0$ at the point $x=1, y=2$.

Solution.

$$x^2+y^2+3xy-11=0$$

$$\frac{dy}{dx} = 2x + 2y\frac{d}{dx} + 3y + 3x\frac{d}{dx}$$

$$\frac{d}{dx}(2y+3x) + 2x + 3y$$

$$\frac{d}{dx}(2y+3x) = -(2x+3y)$$

$$\frac{d}{dx} = -\frac{(2x+3y)}{2y+3x}$$

$$\left.\frac{dy}{dx}\right|_{x=1, y=2} = -\frac{(2(1)+3(2))}{(2(2)+3(1))}$$

$$= \frac{(-2+6)}{(4+3)} = \frac{-8}{7}.$$

$$m_1 = -\frac{8}{7}$$

Equation of the tangent

$$y-y_1 = m(x-x_1)$$

$$y-2 = -\frac{8}{7}(x-1)$$

$$y-2 = -\frac{8x+8}{7}$$

$$7y-14 = -8x+8$$

$$7y+8x-14-8=0$$

$$7y+8x-22=0$$ — Tangent Equation.

$$-7y+8x-22=0$$

$$7y+8x-22=0$$

Equation of the Normal

$$y - y_1 = -\frac{1}{m_1}(x - x_1)$$

$$y - 2 = \frac{1}{\frac{7}{8}}(x - 1)$$

$$y - 2 = \frac{7x - 7}{8}$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0$$

$(8y - 7x - 9 = 0)$ - Normal Equation.