

Olumodeji Tololoba Stephanie

8/04/2020

Almsholt 340

Medicine and Surgery

PTH 104

Question

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

$$y = 3x + 2 \dots \text{equ (i)}$$

$$3y = -x - 9$$

$$y = \frac{-x}{3} - \frac{9}{3}$$

$$y = -x/3 - 3 \dots \text{(ii)}$$

$$m_1 = 3$$

$$m_2 = -1/3$$

Test for perpendicularity;

$$m_1 m_2 = -1.$$

$$3 \times -\frac{1}{3} = -1.$$

Since $m_1 m_2 = -1$, they are perpendicular.

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x}{3} + \frac{7}{3} \dots \text{(i)}$$

$$y = x + 6 + 5$$

$$y = x + 11 \dots \text{(ii)}$$

$$m_1 = 2/3$$

$$m_2 = 1$$

Test for perpendicularity.

$$m_1 m_2 = -1$$

$$\frac{2}{3} \times 1 = 2/3$$

$$\therefore m_1 m_2 \neq -1$$

Therefore the two lines are not perpendicular

3. $x^2 + y^2 + 3xy - 14 = 0$ at the point $x=1, y=2$.

Solution

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3 \left[x \frac{dy}{dx} + y \right] - 0 = 0$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\frac{dy}{dx} = 2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

m at the point $x=1$ and $y=2$

$$m = \frac{dy}{dx} \Big|_{x=1, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3}$$

$$m = \frac{-8}{7} \text{ or } -1 \frac{1}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -8/7 (x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x = 14 + 8$$

$$7y + 8x - 22 = 0 \rightarrow \text{Equation of tangent}$$

$$b) m = \frac{-1}{\frac{-8}{7}}$$

$$m = 7/8$$

$$y - y_1 = \frac{7}{8}(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 9 = 0 \rightarrow \text{Equation of Normal}$$