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$$1) y - 3x - 2 = 0 \text{ and } 3y - x + 9 = 0$$

For the lines to be perpendicular, then $m_1 m_2 = -1$

$$y - 3x - 2 = 0$$

$$y = 3x + 2$$

By comparison with $y = mx + c$

$$m_1 = 3$$

$$3y + x + 9 = 0$$

$$y = -\frac{1}{3}x - 3$$

By comparison with $y = mx + c$

$$m_2 = -\frac{1}{3}$$

$$m_1 m_2 = -1$$

$$3 \times -\frac{1}{3} = -1$$

Since $m_1 m_2 = -1$ then the lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

$$2 \quad 3y - 4 = 2x + 3 \quad \text{and} \quad y - 5 = x + 6$$

$$3y = 2x + 3 + 4 \quad * \text{ Making } y \text{ the subject of formula}$$

$$3y = 2x + 7$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

By comparing with $y = mx + c$
 $m_1 = \frac{2}{3}$

$$By \quad -5 = x + 6 \quad * \text{ Making } y \text{ the subject of formula}$$

$$y = x + 11$$

$m_2 = 1$ By comparing with $y = mx + c$

But for the lines to be perpendicular;

$$m_1 m_2 = -1$$

$$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$m_1 m_2 = -1$$

Hence the lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are
NOT perpendicular

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$$7y + 8x - 22 = 0 \text{ is the equation of tangent}$$

b Equation of normal

$$y - y_1 = -\frac{1}{m} (x - x_1)$$

$$y - 2 = \frac{-1}{-\frac{8}{9}} (x - 1)$$

$$y - 2 = \frac{7}{8} (x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0 \text{ is the equation of normal}$$

$$3. \quad x^2 + y^2 + 3xy - 11 = 0 \quad (x=1, y=2)$$

$$m = \frac{dy}{dx}$$

$$x^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \right) = 0$$

$$2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \right) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m = \frac{dy}{dx} \Big|_{x=1, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$