NAME: EGBORO BETHEL OROWO
DEPARTMENT: PHARMACY
MATRIC NO: 19/MHS11/050
DATE: 06-04-2020

MAT 104 ASSIGNMENT.

QUESTION 1

$$y = \frac{1}{x-2}$$

The function is defined for all real numbers except $x = 2$

Domain: [All real numbers except $x = 2$]
Co-Domain: [All real numbers except $y = 0$]

QUESTION 2

$$K = \ln V$$

$$\frac{dK}{dV} = \frac{1}{V}$$

QUESTION 3

a) $2x - 3y - 2 = 0$

$$-3y = 0 + 2 - 2x$$

$$-3y = -2x + 2$$

$$y = \frac{-2x+2}{-3}$$

$$y = \frac{-(-2x+2)}{3}$$

$$y = \frac{2x-2}{3}$$

b) $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4-x^2}$$

QUESTION 4

$p = \sin^{-1} t$. find derivative of p

Soln

$p = \arcsin(t)$

$p = \frac{t}{\sin}$

$t = \sin p$ --- *

differentiating both sides

$\frac{dt}{dp} = \cos p$

except

but we want $\frac{dp}{dt}$

$\therefore \frac{dp}{dt} = \frac{1}{\cos p}$

$\frac{dp}{dt} = \frac{1}{\sqrt{1-t^2}}$

Recall: $\cos^2 p + \sin^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1-\sin^2 p}$

but $\sin p = t$

$\therefore \cos p = \sqrt{1-t^2}$

QUESTION 5

$f(x) = 2x^2 - 5, \quad g(x) = 4x - 2$

i $fog(x) = f(g(x))$

$= f(4x-2)$

$= 2(4x-2)^2 - 5$

$= 2(4x-2)(4x-2) - 5$

$= 2(16x^2 - 8x - 8x + 4) - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

ii $gof(x) = g(f(x))$

$= g(2x^2 - 5)$

$= 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

QUESTION 6

$f(x) = 3x^2 - 2x + 1$

Show that $f_e(x) + f_o(x) = f(x)$

$$f_e(x) = \frac{f(x) + f(-x)}{2}$$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$
$$= 3x^2 + 2x + 1$$

$$f_e(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$f_e(x) = \frac{6x^2 + 2}{2}$$

$$f_e(x) = \frac{\cancel{2}(3x^2 + 1)}{\cancel{2}} = 3x^2 + 1$$

$$f_o(x) = \frac{f(x) - f(-x)}{2}$$

$$= \frac{(3x^2 - 2x + 1) - (3x^2 + 2x + 1)}{2}$$

$$= \frac{\cancel{3x^2} - 2x + \cancel{1} - \cancel{3x^2} - 2x - \cancel{1}}{2}$$

$$= \frac{-4x}{2}$$

$$f_o(x) = -2x$$

$\therefore f_e(x) + f_o(x) = 3x^2 + 1 - 2x$

$f_e(x) + f_o(x) = 3x^2 - 2x + 1$

$\therefore f_e(x) + f_o(x) = f(x)$

QUESTION 7

$y = \cos x$ from 1st principle. Differentiate.

Soln

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x +$

but $y = \cos x$

$\therefore \delta y = \cos($

Consider from

$\cos(A+B)$

$\cos(A-B)$

$\cos(A+B) - \cos$

compare

Let

$A + B = x$

$A - B = x$

Adding 1)

$2A = 2x$

$A = 2x$

$A = x$

$B = \delta$

compare

$\cos(x + \delta$

$\frac{\delta y}{\delta x} =$

$\frac{\delta y}{\delta x} =$

$\frac{\delta y}{\delta x} =$

A stand

lim

$\delta x \to$

Fin

$\lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

$\delta y = \cos(x + \delta x) - y$

but $y = \cos x$

$\therefore \delta y = \cos(x + \delta x) - \cos x$ --- *

Consider from Trig.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$- \quad \cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$ --- (2*)

compare * and 2*

Let

$A + B = x + \delta x$ -(1)

$A - B = x$ -- (11)

Adding 1) and 11)

$2A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$$\left.\begin{aligned} A &= x + \frac{\delta x}{2} \\ B &= \frac{\delta x}{2} \end{aligned}\right\} (3*)$$

compare * and 2*

$\cos(x + \delta x) - \cos x = -2 \sin\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right)$

$\frac{\delta y}{\delta x} = \frac{-2 \sin(x + \delta x/2) \sin(\delta x/2)}{\delta x}$

$\frac{\delta y}{\delta x} = \frac{-\sin(x + \delta x/2) \sin(\delta x/2)}{\delta x/2}$

$\frac{\delta y}{\delta x} = -\sin(x + \delta x/2) \frac{\sin(\delta x/2)}{\delta x/2}$ -(4*)

A standard limit

$\lim_{\delta x \to 0} \frac{\sin(\delta x/2)}{\delta x/2} = 1$

Finding limit of (4*) as $\delta x \to 0$

$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin\left(x + \frac{\delta x}{2}\right) \frac{\sin(\delta x/2)}{\delta x/2}$

$= -\sin(x + 0) \cdot 1$

$= -\sin x$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

Hence, if $y = \cos x$, then $\frac{dy}{dx} = -\sin x$.

QUESTION 8

Find $\frac{dy}{dx}$; if $y = 3t^2$, $x = \frac{1}{t^2}$

Soln

$y = 3t^2$ $\qquad$ $x = 1/t^2 = t^{-2}$

$\frac{dy}{dt} = 6t$ $\qquad$ $\frac{dx}{dt} = -2t^{-3}$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= 6t \div -2t^{-3}$$

$$= \frac{6t}{-2t^{-3}}$$

$$\frac{dy}{dx} = \cancel{-3t^4} \quad -3t^4$$

QUESTION 9

$\frac{dy}{dx}$ if $y = x^2 \cos 2x \, e^{4x}$

$$\ln y = \ln (x^2 \cos 2x \, e^{4x})$$

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2} - \frac{2\sin 2x}{\cos 2x} + \frac{4e^{4x}}{e^{4x}}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$$

multiplying b

$\frac{dy}{dx} = y\left(\frac{2}{x}\right.$

But $y = x^2$ (

$\therefore \frac{dy}{dx} = x^2$ (

QUESTION

$y = \sin($

Solution

Let $u =$

$y =$

$\frac{dy}{dx} = \frac{dy}{du}$

$= \cos$

$= 9x$

but, $u =$

$\therefore \frac{dy}{dx} =$

multiplying both sides by y, we have

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

But, $y = x^2 \cos 2x\, e^{4x}$

$$\therefore \frac{dy}{dx} = x^2 \cos 2x\, e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

QUESTION 10

$y = \sin(3x^3 + 5)$, find dy/dx

Solution

Let $u = 3x^3 + 5$, $du/dx = 9x^2$

$y = \sin u$, $dy/du = \cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

but, $u = 3x^3 + 5$

$$\therefore \frac{dy}{dx} = 9x^2 \cos(3x^3 + 5)$$