Moses Dreye Telphy

1.

Examine whether or not these pair of lines are perpendicular to each other

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

1. $y - 3x - 2 = 0$ - Line A

$3y + x + 9 = 0$ - Line B

1.) $y = 3x + 2$ $\quad m = 3$ (A)

$\frac{3y}{3} = \frac{-x}{3} - \frac{9}{3}$

$y = \frac{-1}{3}x - 3 \quad m = -1/3$ (Line B)

∴ Line A and Line B are perpendicular to each other because the slope of Line B is the negative inverse of slope A

2) $3y - 4 = 2x + 3$ - (Line A) , $y - 5 = x + 6$ - (Line B)

$3y = 2x + 3 + 4$ $\quad y = x + 6 + 5$

$\frac{3y}{3} = \frac{2x + 7}{3} \quad y = x + 11$

$y = \frac{2}{3}x + 7/3$ $\quad$ ∵ $m = 1$

$m = 2/3$ $\quad$ ∴ Line A and Line B are not perpendicular to each other because neither Line B nor Line A are the negative inverse slope.

3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at point $(1, 2)$ $(x, y)$

$x^2 + y^2 + 3xy = 11$

$\frac{dy}{dx} \frac{d}{dx}(x^2 + y^2 + 3xy) = \frac{d}{dx}(11)$

$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dy} - \frac{3d(xy)}{dx} = \frac{d}{dx}(11)$$

$$2x + 2y\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 0$$

$$2x - 3y = 3x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$2x - 3y = \frac{dy}{dx}(3x - 2y)$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 2y}$$

$$m = \frac{2(1) - 3(2)}{3(1) - 2(2)} = \frac{-4}{-1}$$

~~$m = 2(1) - 3($~~ $m = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{2x - 3y}{3x - 2y}(x - 1)$$

~~$y - 2 = 2x - 2(1)x - 3(2)$~~ $m = \frac{dy}{dx} = \frac{2(1) - 3(2)}{3(1) - 2(2)}$

$3(1) - 2(2)$

$$y = \frac{2 - 6}{3 - 4}x + 2 \qquad m = \frac{2 - 6}{3 - 4}$$

$$y = \frac{-4}{-1}x + 2$$

$$y = 4x + 2$$

$$y = \frac{2x - 3y}{3x - 2y}(x - 1) + 2$$

$$y = \frac{2x^2 - 3xy}{3x - 2y} - \frac{2x - 3y}{3x - 2y} + 2$$

$$y = 2x^2$$

a) Equation of tangent

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 4(x - 1)$$
$$y - 2 = 4x - 4$$
$$y = 4x - 4 + 2$$
$$y = 4x - 2$$

b) Equation of normal to the curve

$$m_{normal} = -1 / m_{tangent}$$
$$m_{normal} = -1/4$$
$$y - y_1 = m'(x - x_1)$$
$$y - 2 = -1/4 (x - 1)$$
$$y - 2 = -1/4 x + 1/4$$
$$y = -1/4 x + 1/4 + 1/2$$
$$y = -1/4 x + 3/4$$