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100 LEVEL
Matric No: 19/EHG06/017
Math 102.

1. $A = 2i - j$, $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$

i) $-3A + 7B - 8C$
ii) If $k = 2A + 4B - C$, find the direction cosine of k
iii) ~~$A \times (B \times$~~ $A \times (B \times C)$
iv) $(3A \times B)\ (A \times 2B)$
v) $A - 2B - C$

2. Define perpendicular and Coplanar vectors

Solutions

i) $-3(2i - j) + 7(3i - j - 11k) - 8(4i + 4j - 5k)$
$-6i + 3j + 21i - 7j - 77k - 32i - 32j + 40k$
$-6i + 21i - 32i + 3j - 7j - 32j - 77k + 40k$
~~$= -17i$~~
$= -17i - 36j - 37k$

ii) $k = 2A + 4B - C$
$k = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$
$k = 4i - 2j + 12i + 4j - 44k - 4i + 4j - 5k$
$k = 4i + 12i - 4i - 2j + 4j + 4j - 44k - 5k$
$k = 12i + 6j - 49k$

The direction cosine of k.

$k = 12i + 6j - 49k$

$|k| = \sqrt{12^2 + 6^2 + (-49)^2}$

$= \sqrt{144 + 36 + 2401}$

$= \sqrt{2906}$

$= 53.91$

$\cos\alpha = \frac{a_x}{|k|}$ Let $a_x = 12$

$\cos\beta = \frac{a_y}{|k|}$ $a_y = 6$

$\cos\gamma = \frac{a_z}{|k|}$ $a_z = -49$

$\cos\alpha = \frac{12}{53.91} = 0.2226$

$\cos\beta = \frac{6}{53.91} = 0.1113$

$\cos\gamma = \frac{49}{53.91} = 0.9089$

~~$\alpha = 77°$, $\beta =$~~ $\alpha = 77.19$

$\beta = 84$

$\cos\alpha = 0.2226$

$\cos\beta = 0.1113$

~~$\cos\gamma$~~ $\cos\gamma = 0.9089$

iii) $A \times (B \times C)$

$B \times C = \begin{vmatrix} + & - & + \\ i & j & k \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$

$= i(-5+44) - j(-15+44) + k(12-4)$

$= 39i - 29j + 8k$

$A \times (B \times C) = \begin{vmatrix} + & - & + \\ i & j & k \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$

$= i(-8+0) - j(16-0) + k(-58+39)$

$= -8i - 16j + (-19k)$

~~$= -8i - 16$~~ $= -8i - 16j - 19k$ //

iv) $(3A \times B) \cdot (A \times 2B)$

$(3(2i-j) \times (3i+j-11k))$

$(6i - 3j) \times (3i+j-11k)$

$\begin{vmatrix} + & - & + \\ i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$

$= i(33-0) - j(-66-0) + k(-9-6)$

$-32i + 69j - 15k = 33i + 66j - 15k$

A × 2B

$(2i - j) \times (2(3i + j - 11k))$

$(2i - j) \times (6i + 2j - 22k)$

$$= \begin{vmatrix} i & j & k \\ 1 & -1 & 6 \\ 6 & 2 & -22 \end{vmatrix}$$

$= i(22 - 12) - j(-44 - 0) + k(4 + 6)$

$= 22i + 44j + 10k$

$(33i + 66j - 15k) \cdot (22i + 44j + 10k)$

~~726i +~~

$726 + 2904 - 150 = 3480$

v) $A - 2B - C$

$(2i - j) - (2(3i + j - 11k)) - (4i + 4j - 5k)$

~~$2i - j - 6i - 2j + 22k - 4i + 4j$~~

$2i - j - 6i - 2j + 22k - 4i - 4j + 5k$

$= 2i - 6i - 4i - j - 2j - 4j + 22k + 5k$

$= -8i - 7j + 27k$

$= -8i - 7j + 27k$

2) Perpendicular Vectors

A Two vectors are said to be perpendicular if their angle is a right angle and their scalar product is zero

ii) Coplanar Vectors

Vectors are said to be coplanar when they are parallel to the same plane or lie on the same plane. The scalar triple product of three coplanar vectors is equal to zero.