DARE BENEDICT OLUBUNDLA
MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO: 111
MAT 102 ASSIGNMENT (Mrs. Funmilayo Baba)

If $A = 2i - j$, $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$ find the following;

(i) $-3A + 7B - 8C$

(ii) if $k = 2A + 11B - C$, find the direction cosine of k

(iii) $A \times (B \times C)$

(iv) $(3A \times B) \cdot (A \times 2B)$

(v) $A - 2B - C$

Solution

$\underline{A} = 2i - j$

$\underline{B} = 3i + j - 11k$

$\underline{C} = 4i + 4j - 5k$

(i) $-3A + 7B - 8C$

$= -3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k)$

$= -6i + 3j + 21i + 7j - 77k - 32i - 32j + 40k$

$= -6i + 21i - 32i + 3j + 7j - 32j - 77k + 40k$

$= -17i - 22j - 37k$

(ii) $K = 2A + 4B - C$

$K = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$

$K = 4i - 2j + 12i + 4j - 44k - 4i - 4j + 5k$

$K = 4i + 12i - 4i - 2j + 4j - 4j - 44k + 5k$

$K = 12i - 2j - 39k$

$\therefore |K| = \sqrt{(12)^2 + (-2)^2 + (-39)^2}$

$= \sqrt{144 + 4 + 1521}$

$= \sqrt{1669}$

$= 40.85$

$\Rightarrow l = \cos\alpha = \frac{12}{\sqrt{1669}}$

$m = \cos\beta = \frac{-2}{\sqrt{1669}}$

$n = \cos\gamma = \frac{-39}{\sqrt{1669}}$

(iii) $A \times (B \times C)$:

$$(B \times C) = \begin{vmatrix} i & j & K \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$i\begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - j\begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$$

$i[(-5 \times 1) - (4 \times -11)] - j[(-5 \times 3) - (4 \times -11)] + k[(3 \times 4) - (4 \times 1)]$

$= 39i - 29j + 8k$

∴ For

$A \times (B \times C)$;

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$$

$$i \begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 39 & -29 \end{vmatrix}$$

$i[(8 \times -1) - (-29 \times 0)] - j[(2 \times 8) - (39 \times 0)] + k[(2 \times -29) - (39 \times -1)]$

$\therefore A \times (B \times C) = -8i - 16j - 19k$

(iv) $(3A \times B) \cdot (A \times 2B)$

$(3A \times B) = 3(2i - j) \times (3i + j - 11k)$

$= (6i - 3j) \times (3i + j - 11k)$

$$\therefore (3A \times B) = \begin{vmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$i \begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - j \begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + k \begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$i[(-11 \times -3) - (1 \times 0)] - j[(-11 \times 6) - (3 \times 0)] + k[(6 \times 1) - (3 \times -3)]$

$\Rightarrow (3A \times B) = 33i + 66j + 15k$

$(A \times 2B) = (2i - j) \times 2(3i + j - 11k)$

$= (2i - j) \times (6i + 2j - 22k)$

$$\therefore (A \times 2B) = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$i\begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$i[(-22 \times -1) - (2 \times 0)] - j[(-22 \times 2) - (6 \times 0)] + k[(2 \times 2) - (6 \times -1)]$$

$$\Rightarrow (A \times 2B) = 22i + 44j + 10k$$

$$\therefore (3A \times B) \cdot (A \times 2B) = (33i + 66j + 15k) \cdot (22i + 44j + 10k)$$

$$(3A \times B) \cdot (A \times 2B) = 726 + 2904 + 150$$

$$(3A \times B) \cdot (A \times 2B) = 3780$$

(v) $A - 2B - C$

$$= (2i - j) - 2(3i + j - 11k) - (4i + 4j - 5k)$$

$$= 2i - j - 6i - 2j + 22k - 4i - 4j + 5k$$

$$= 2i - 6i - 4i - j - 2j - 4j + 22k + 5k$$

$$= -8i - 7j + 27k$$

2. Define Perpendicular and Co-planar vectors.

Answer;

Perpendicular Vectors, Two vectors A & B are said to be perpendicular if their scalar product is equal to

zero. ie $A \cdot B = 0$

Co-planer Vectors; Three vectors A, B & C are said to be Co-planer, if their scalar triple product is equal to zero. ie $A \cdot (B \times C) = 0$