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MAT NO: 19/MHS01/274 DEPT: MEDICINE AND SURGERY

MAT 104 ASSIGNMENT

Examine whether or not these pair of lines are perpendicular to each other

i) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

3) Find the equations of the tangent and normal to the axis $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

SOLUTION.

1) $y - 3x - 2 = 0$

$3y + x + 9 = 0$

for the lines to be perpendicular $m_1 m_2 = -1$

$y - 3x - 2 = 0$

making y the subject of the formula

$y = 2 + 3x$ or $y = 3x + 2$

Comparing the equation above with the standard formula $y = mx + c$

$m = 3 \therefore m_1 = 3$

$3y + x + 9 = 0$

making y the subject of formula

$3y = -x - 9$

$y = \frac{-x}{3} - \frac{9}{3}$

$y = -1/3x - 3$

using $y = mx + c$ $m = -1/3 \therefore m_2 = -1/3$

For perpendicularity $m_1 m_2 = -1$

$3 \times \frac{-1}{3} = -1$

$\therefore$ The lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular.

2-) $3y - 4 = 2x + 3$

$y - 5 = x + 6$

for perpendicularity $m_1 m_2 = -1$.

$3y - 4 = 2x + 3$

making y the subject of formula

$3y = 2x + 7$

$y = \frac{2}{3}x + 7/3$

Comparing with $y = mx + c$

$m = 2/3 \therefore m_1 = 2/3$

$y - 5 = x + 6$

making y the subject of formula

$y = x + 11$

Comparing with $y = mx + c$

$m = 1 \therefore m_2 = 1$

$m_1 \times m_2 = -1$ for perpendicularity

$m_1 m_2 = \frac{2}{3} \times 1 = 2/3$

$\therefore$ The lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular.

3) $x^2+y^2+3xy-11=0$ at points $x=1, y=2$.

$m=\frac{dy}{dx}$

$\frac{dy}{dx} = 2x+2y\frac{dy}{dx}+3\left[x\cdot\frac{dy}{dx}+y\cdot 1\right]=0$

$2x+2y\frac{dy}{dx}+3\left(x\frac{dy}{dx}+y\right)=0$

$2x+2y\frac{dy}{dx}+3x\frac{dy}{dx}+3y=0$

Collecting like terms

$2y\,dy/dx+3x\,dy/dx=-2x-3y$

$\frac{dy}{dx}(2y+3x)=-2x-3y$

$\frac{dy}{dx}=\frac{-2x-3y}{2y+3x}$

$m=\frac{dy}{dx}\Big|_{x=1,y=2}$

$=\frac{-2x-3y}{2y+3x} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3} = -\frac{8}{7}$

$m_1=-8/7$

EQUATION OF TANGENT

$y-y_1=m(x-x_1)$

$y-2=\frac{-8}{7}(x-1)$

$7(y-2)=-8(x-1)$

$7y-14=-8x+8$

$7y+8x-14-8=0$

$7y+8x-22=0$ {Equation of tangent}

EQUATION OF NORMAL

$m_1 \times m_2 = -1$

$m_2 = -1 / m_1$

$m_2 = -1 \div -\frac{8}{7}$

$m_2 = +1 \times \frac{7}{+8} = 7/8$

$y - y_1 = m_2(x - x_1)$

$y - 2 = \frac{7}{8}(x - 1)$

$8(y - 2) = 7(x - 1)$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$

$8y - 7x - 9 = 0$ {Equation of the normal}