Name: Ezekwueche Nunachukwu Belwolisa.
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Question
Examine whether or not these pair of lines are perpendicular to each other.

① $y - 3x - 2 = 0$ (i) and $3y + x + 9 = 0$ (ii)

$y = mx + c$ [Equation of a straight line]

$y = 3x + 2$

For (i) $y = mx + c$

$m_1 = 3$

For (ii) $3y + x + 9 = 0$

$3y = -x - 9$

$y = \frac{-x}{3} - \frac{9}{3}$

$y = \frac{-x}{3} - 3$

$y = mx + c$

$m_2 = -1/3$

For m_1 and m_2 to be perpendicular, their product must give -1

$m_1 m_2 = -1$

$3 \times -1/3 = -1$

∴ So therefore, they are perpendicular to each other.

② 3y

(i)

(ii)

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

$y = mx + c$

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2x}{3} + \frac{7}{3}$

$y = mx + c$

$m_1 = 2/3$

For (ii) $y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$

$y = mx + c$

$m_2 = 1$

$m_1 m_2 = -1$

$\frac{2}{3} \times 1 = 2/3$

It doesn't give -1 so therefore, they aren't perpendicular.

(3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

Differentiate $x^2 + y^2 + 3xy - 11 = 0$.

$2x + 2y\,dy/dx + 3(x\,dy/dx + y) - 0 = 0$.

$2x + 2y\,dy/dx + 3x\,dy/dx + 3y = 0$.

Make dy/dx the subject of the formula.

$2y\,dy/dx + 3x\,dy/dx = -3y - 2x$

$\frac{dy}{dx}(2y + 3x) = -3y - 2x$

$\frac{dy}{dx} = \frac{-3y - 2x}{2y + 3x}$

$$m = \frac{dy}{dx}\Big|_{\substack{x=1\\y=2}} = \frac{-3(2)-2(1)}{2(2)+3(1)} = \frac{-6-2}{4+3}$$

$$m = -8/7.$$

For equation of the tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x-1)$$

multiply through by 7.

$$7y - 14 = -8(x-1)$$

$$7y - 14 = -8x + 8$$

$$7y = -8x + 8 + 14$$

$$7y = -8x + 22$$

$$7y + 8x - 22 = 0$$

For Equation of the normal

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - y_1 = \frac{7}{8}(x - x_1)$$

$$y - 2 = \frac{7}{8}(x-1)$$

multiply through by 8

$$8y - 16 = 7(x-1)$$

$$8y - 16 = 7x - 7$$

$$8y = 7x - 7 + 16$$

$$8y = 7x + 9$$

$$8y - 7x - 9 = 0$$