10 $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$ $\qquad du/dx = 9x^2$

$y = \sin u$ $\qquad dy/du = \cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u.$$

but $u = 3x^3 + 5$

$$\frac{dy}{dx} = 9x^2 \cos(3x^3 + 5)$$

$$= 9x^2 \cos(3x^3 + 5)$$

If $y =$

(a) dy/dx.

$$\text{If } y = x^2 \cos 2x \, e^{4x}$$

$$\ln y = \ln(x^2 \cos 2x \, e^{4x})$$

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$$

multiplying both sides by y, we have

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right) \text{ but } y = x^2 \cos 2x \, e^{4x}$$

$$\frac{dy}{dx} = x^2 \cos 2x \, e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right).$$

8. If $y = 3t^2$

$x = \frac{1}{t^2}$

find $\frac{dy}{dx}$

$y = 3t^2$ $\quad dy/dt = 6t$

$x = \frac{1}{t^2}$ $\quad dx/dt = -2t^{-3} = -\frac{2}{t^3}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{\frac{-2}{t^3}} = 6t \times \frac{t^3}{-2}$$

$$= -3t^4$$

$\therefore dy/dx = -3t^4$

$$\cos(x + dx) = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$\delta y = 2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$$

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \delta x/2\right)\sin(\delta x/2)}{\delta x}$$

$$\frac{\delta y}{\delta x} = -\sin\left(x + \delta x/2\right)\frac{\sin\left(\delta x/2\right)}{\left(\delta x/2\right)}$$

$$= -\sin\left(x + dx/2\right)\frac{\sin \delta x/2}{\delta x/2}.$$

$$\lim_{dx \to 0} \frac{\sin(\delta x/2)}{\delta x/2} = 1$$

$$\lim_{dx \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin\left(x + \delta x/2\right)\frac{\sin\left(\delta x/2\right)}{\left(\delta x/2\right)}$$

$$= -\sin(x)$$

$$= -\sin(x)$$

$$\lim_{\delta x \to x} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

7. $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x + \delta x) - y$

$y = \cos x$

$\delta y = \cos(x + \delta x) - \cos x \quad \ldots (1)$

Consider from trigonometry.

$(\cos A + B) = \cos A \cos B - \sin A \sin B$

$(\cos A - B) = \cos A \cos B + \sin A \sin B$

$\cos(A + B) - (\cos A - B)$

$= -2 \sin A \sin B \quad \ldots (2*)$

Compare equ 1 and 2nd

let

$A + B = x + \delta x \quad \ldots (i)$

$A - B = x \quad \ldots (ii)$

Adding (i) and (ii)

$2A = 2x + \delta x$

$A = \dfrac{2x + \delta x}{2}$

$$\left.\begin{aligned} A &= x + \frac{\delta x}{2} \\ B &= \frac{\delta x}{2} \end{aligned}\right\} 3$$

$\therefore f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

$/dx$.

$y = x^2 \cos^3 2x\, e^{4x}$

$y = \ln(x^2 \cos 2x\, e^{4x})$

6. $f(x) = 3x^2 - 2x + 1 = 0$

$f_e(x) + f_o(x) = f(x)$

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$= f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$

$= \frac{3x^2 + 3x^2 - 2x + 2x + 2}{2}$

$= \frac{6x^2 + 2}{2}$

$= \frac{2(3x^2 + 1)}{2}$

$= 3x^2 + 1.$

$f_o = \frac{f(x) - f(-x)}{2}$

$= \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \frac{-4x}{2} = -2x$

(9)

5) If $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

(a) $fog(x)$

(b) $gof(x)$

(a) $(fog)x$

$f(go)$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$$
\begin{aligned}
(fog)x &= f(g(x)) \\
&= f(4x-2) \\
&= 2(4x-2)^2 - 5 \\
&= 2(4x-2)(4x-2) - 5 \\
&= 2(16x^2 + 4 - 8x - 8x) - 5 \\
&= 32x^2 - 32x + 8 - 5 \\
&= 32x^2 - 32x + 3.
\end{aligned}
$$

b) $gof(x)$

$g(f(x))$

$g(2x^2 - 5)$

$4(2x^2 - 5) - 2 = 8x^2 - 20 - 2$

$= 8x^2 - 22$

MATHS

4) If $p = \sin^{-1} t$ find the derivative of p

$p = \frac{t}{\sin p}$

$t = \sin p$ - - - (1)

Recall that $\sin^2 p + \cos^2 y = 1$ - - - (2)

$\frac{dt}{dp}$ of (1) $= \cos p$

From (2) $\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin^2 p}$

$\cos p = \sqrt{1 - t^2}$

$\frac{dt}{dp} = \cos p = \sqrt{(1+t)^2}$

$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$

1. $y = \frac{1}{x-2}$

It is not defined because is a fraction because of the denominator.
The function is defined for all real numbers except $x = 2$
Domain - Real numbers except $x = 2$
co-domain = Real numbers except $y = 0$

2. If $k = \ln v$ differentiate k

$$\frac{dk}{dv} = \frac{1}{v}$$

3. $2x - 3y - 2 = 0$

$$2x - 3y = +2$$

$$\frac{-3y}{-3} = \frac{2 - 2x}{-3}$$

$$y = \frac{-2 + 2x}{3}$$