$y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

for them to be perpendicular to each other

$m_1 m_2 = -1$

$\therefore y - 3x - 2 = 0$ ... equ ①

$\therefore y = 3x + 2$

Recall $- y = mx + c$

$m = 3$

eq -- ②

$3y + x + 9 = 0$

$\therefore \frac{3y}{3} = \frac{-x - 9}{3}$ divide through by 3

$y = \frac{-x}{3} - 3$

$\therefore$ recall $y = mx + c$

$\Rightarrow m_2 = -1/3$

$\therefore m_1 m_2 = -1$

$\cancel{3}(-\frac{1}{\cancel{3}}) = -1$

$\therefore$ The lines are perpendicular to each other.

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

$\therefore$ using equ ①

$3y - 4 = 2x + 3$

$\Rightarrow 3y = 4 + 2x + 3$

$$\frac{3y}{3} = \frac{2x+7}{3}$$ divide through by 3

$$y = \frac{2x}{3} + \frac{7}{3}$$

$\therefore$ Recall $y = mx + c$

$\therefore m_1 = \frac{2}{3}$

Using $y - 5 = x + 6$

$y - 5 = x + 6$

$\therefore$ making y subject formula

$y = x + 11$

$\therefore$ Recall $y = mx + c$

$\therefore m_2 = 1$

$\therefore \quad m_1 m_2 \neq -1$

$\frac{2}{3}(1) \neq -1$

The lines are not perpendicular to each other

3 find the equation of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1$

Solution

$x^2 + y^2 + 3xy - 11 = 0$

$$\therefore \quad 2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2x + 3y + \left(2y\frac{dy}{dx} + 3x\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(2y + 3x\right) = -(2x + 3y)$$

$$\therefore \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

$$\therefore m = \frac{dy}{dx}\Big| = \frac{-2x - 3y}{2y + 3x} = -\frac{8}{7}$$

$$\therefore y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$7y + 8x - 22 = 0$ which equal the new tangent

$\therefore$ for equation at normal

$$m = \frac{-1}{-\frac{8}{7}} = \frac{7}{8}$$

$$\therefore y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$\cancel{8y + 7x}$

$$8y - 7x - 9 = 0$$

which gives the equation at normal