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MAT 102

Mechanical Engr.

S/N: 37

1) $-3A + 7B - 8C$

$$\begin{aligned} &= -3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k) \\ &= -6i + 3j + 21i + 7j - 77k - 32i - 32j + 40k \\ &= -17i - 22j - 37k \end{aligned}$$

11) $K = 2A + 4B - C$ find direction cosine

~~$K = 2A$~~ $K = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$

$$K = 4i - 2j + 12i + 4j - 44k - 4i - 4j + 5k$$

1. ~~$K = 12i + 6j - 49k$~~ $K = 12i - 2j - 39k$

$$|K| = \sqrt{(12)^2 + (-2)^2 + (-39)^2}$$

$$|K| = \sqrt{1669} = 40.85$$

$$|K| = \sqrt{144 + 4 + 1521}$$

~~$|K| = \sqrt{2581} = 50.80$~~

$$l = \cos \alpha = \frac{12}{40.85}$$

$$\alpha = \cos^{-1}\left(\frac{12}{40.85}\right)$$

$$\alpha = \cos^{-1}(0.2938)$$

~~$\alpha = 72.91^\circ$~~ $\alpha = 72.91^\circ$

$$m = \cos \beta = \frac{-2}{40.85}$$

$$\beta = \cos^{-1}\left(\frac{-2}{40.85}\right)$$

$$\beta = \cos^{-1}(0.1469)$$

~~$\beta = 83.22^\circ$~~ $\beta = 83.22^\circ$

$$n = \cos \gamma = \frac{-39}{40.85}$$

$$\gamma = \cos^{-1}\left(\frac{-39}{40.85}\right)$$

$$\gamma = \cos^{-1}(-1.1995)$$

$$\gamma = 17.31^\circ$$

III) $A \times (B \times C)$

$$(B \times C)_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$$

$$\hat{i}(-5 - (-44)) - \hat{j}(-15 - (-44)) + \hat{k}(12 - 4)$$

$$\hat{i} 39 + 29\hat{j} + 8\hat{k}$$

So $A \times (B \times C)_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 39 & 29 & 8 \end{vmatrix}$

$$= \hat{i} \begin{vmatrix} -1 & 0 \\ 29 & 8 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -1 \\ 39 & 29 \end{vmatrix}$$

$$= -8\hat{i} - 16\hat{j} + \hat{k}(58 - (-39))$$

$$= -8\hat{i} - 16\hat{j} + 97\hat{k}$$

$$\text{IV) } 3A = 3(2\hat{i} - \hat{j}) = 6\hat{i} - 3\hat{j} \quad 2B = 2(3\hat{i} + \hat{j} - 11\hat{k}) = 6\hat{i} + 2\hat{j} - 22\hat{k}$$

$$(3A \times B) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -3 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} -3 & 0 \\ 2 & -22 \end{vmatrix} - \hat{j} \begin{vmatrix} 6 & 0 \\ 6 & -22 \end{vmatrix} + \hat{k} \begin{vmatrix} 6 & -3 \\ 6 & 2 \end{vmatrix}$$

$$= 66\hat{i} + 132\hat{j} + \hat{k}(12 - (-18))$$

$$= 66\hat{i} + 132\hat{j} + 30\hat{k}$$

$$(A \times 2B) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix} \quad (-)$$

$$22\hat{i} + 44\hat{j} + 10\hat{k}$$

$$(3A \times B) \cdot (A \times 2B)$$

$$(66\hat{i} + 132\hat{j} + 30\hat{k}) \cdot (22\hat{i} + 44\hat{j} + 10\hat{k})$$

$$= 1452\hat{i} + 5808\hat{j} + 300\hat{k} //$$

$$\begin{aligned} \text{v)} \quad & A - 2B - C \\ & = 2i - j - 2(3i + j - 11k) - (4i + 4j - 5k) \\ & = 2i - j - 6i - 2j + 22k - 4i - 4j + 5k \\ & = -8i - 7j + 27k \end{aligned}$$

2) i) Vectors are said to be coplanar when their scalar triple product is zero i.e. $a \cdot (b \times c) = 0$.

ii) Two vectors A and B are perpendicular if and only if their scalar product is equal to zero.