3 $x^2+y^2+3xy-11=0$ at $(1,2)$

$= x^2 + y(y+3x) - 11 = 0$

$y(y+3x) = 11 - x^2$

$\therefore y = \frac{11-x^2}{y+3x}$ at $(1,2)$

$\frac{dy}{dx} = \frac{V\, du/dx - u\, dv/dx}{V^2}$

$u = 11 - x^2$; $du/dx = -2x$

$v = y + 3x^2$; $dv/dx = 1 + 3 = 4$

$\frac{dy}{dx} = \frac{(y+3x)(-2x) - (11-x^2)(4)}{(y+3x)^2}$

$= \frac{-2xy - 6x^2 - [44 - 4x^2]}{y^2 + 6xy + 9x^2}$

$= \frac{-2xy - 6x^2 - 44 + 4x^2}{y^2 + 6xy + 9x^2}$

$= \frac{-2xy - 2x^2 - 44}{y^2 + 6xy + 9x^2}$

$\frac{dy}{dx} = -\frac{(2xy + 2x^2 + 44)}{y^2 + 6xy + 9x^2}$

$\left.\frac{dy}{dx}\right|_{x=1 \text{ and } y=2} = -\frac{(2(1)(2) + 2(1)^2 + 44)}{2^2 + 6(1)(2) + 9(1)^2} = -2$

$m = -2$

$x_1 = 1,\ y_1 = 2$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 1)$

$y - 2 = -2x - 2$

$y + 2x - 4 = 0$

Equation of normal

$y - y_1 = -1/m(x - x_1)$

$y - 2 = +1/2(x - 1)$

$2y - 4 = x - 1$

$2y - x - 3 = 0$

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Let $A = y - 3x - 2 = 0$

$= \frac{dy}{dx} - 3 - 0 = 0$

$= \frac{dy}{dx} - 3 = 0$

$= \frac{dy}{dx} = 3$

Let $B = 3y + x + 9 = 0$

$3\frac{dy}{dx} + 1 + 0 = 0$

$3\,dy/dx + 1 = 0$

$dy/dx = -1/3$

$A + B$

i.e $y - 2x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Let $A - 3y - 4 = 2x + 3$

$3\frac{dy}{dx} - 0 = 2 + 0$

$3\,dy/dx = 2$

$dy/dx = 2/3$

Let $b = y - 5 = x + 6$

$dy/dx - 0 = 1 + 0$

$dy/dx = 1$

$\therefore A \neq B$

i.e $3y - 4 = 2x + 3$ and $y - 5 = x + c$ is not perpendicular.

NAME: Jacoob Deinma Praise
MATRIC NO: 19/MHS01/214
COLLEGE: MHS
DEPARTMENT: Mbbs
COURSE: Math 104

Question
Examine whether or not these pair of lines are perpendicular to each other.

0 $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

1 $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

2 Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at point $x = 1, y = 2$

Solution

$x^2 + y^2 + 3xy - 11 = 0$