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MATRIC NO: 19/MHS06/007
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1) For what values of x is the function $y = \frac{1}{(x-2)}$ defined? State the domain and codomain

Solution

In $y = \frac{1}{x-2}$, the quantity $x-2$ cannot be equal to zero

Thus

$x \neq 2$

Hence $\quad \longleftarrow (\infty, 2) \underset{2}{\bullet} (2, \infty) \longrightarrow$

Domain $= (-\infty, 2) \cup (2, \infty)$

For $x = 2 + \frac{1}{n}$ where n is any non zero real number

$y = \frac{1}{2+\frac{1}{n}-2} = n$

y can be any real number expect 0, so the range is $(\infty, 0) \cup (0, \infty)$

2) If $k = \ln Y$, differentiate k

Solution

$k = \ln Y = \Rightarrow e^k$

differentiated wrt k

$\frac{dk}{dv} = e^y$

$\therefore \frac{dv}{dk} = \frac{1}{e^k}$ recall $e^x = Y$

Thus

$\frac{dv}{dk} = \frac{1}{Y}$ //

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9) Find dy/dx if $y = x^2 \cos 2x\ e^{4x}$.

Solution

Product rule of three function

if $y = f(x)\ g(x)\ h(x)$

$\frac{dy}{dx} = f'(x)\ g(x)\ h(x) + f(x)\ g'(x)\ h(x) + f(x)\ g(x)\ h'(x)$

Let $f(x) = x^2,\ f'(x) = 2x$

$g(x) = \cos 2x,\ g'(x) = -2\sin(2x)$

$h(x) = e^{4x},\ h'(x) = 4e^{4x}$

Thus

$\frac{dy}{dx} = 2x\cos 2x\, e^{4x} + x^2[-2\sin 2x]\, e^{4x} + x^2 \cos 2x\,[4e^{4x}]$ //

10) Given that $y = \sin(3x^3 + 5)$ find the derivative of y

Solution

Let $v = 3x^3 + 5$

Thus $y = \sin v$

$\frac{dv}{dx} = 9x^2$

$\frac{dy}{dv} = -\cos u$

$dy/dx = dv/dx \times dy/dv$

$= 9x^2 \times -\cos v$

$= -9x^2 \cos v$

recall $v = 3x^2 + 5$

$dy/dx = -9x^2 \cos[3x^3 + 5]$

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5) If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $fog(x)$ and $gof(x)$

Solution

$(fog)(x) = f(g(x))$
$= f(4x-2)$
$= 2(4x-2)^2 - 5$
$= 2(16x^2 - 16x - 4) - 5$
$= 32x^2 - 3x^2 - 8 - 5$
$= 32x^2 - 3x^2 - 13$

$(gof)(x) = g(f(x))$
$= g(2x^2 - 5)$
$= 4(2x^2 - 5) - 2$
$= 8x^2 - 20 - 2$
$= 8x^2 - 22$

6) If $f(x) = 3x^2 - 2x + 1 = 0$ Show that $f_e(x) + f_o(x) = f(x)$

$fe(x) = \frac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 = 2(-x) + 1$

$f(-x) = 3x^2 + 2x + 1$

$fe(x) = \frac{3x^2 - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2}$

$= \frac{6x^2 + 2}{2}$

$= \frac{\cancel{2}(3x^2 + 1)}{\cancel{2}}$

$\therefore fe(x) = 3x^2 + 1$

$fo(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$fo(x) = \frac{3x^2 - 2x + 1 - 3x^2 - 2x}{2}$

$\therefore fo(x) = \frac{-4x}{2} = -2x$

$\therefore f(x) = fe(x) + fo(x)$

$f(x) = 3x^2 + 1 - 2x$

$f(x = 3x^2 + 2x + 1$

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7) Differentiate $y = \cos x$ from first principle.

Solution

$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, where $h = \delta x$

Substitute our function $\cos x$

$\lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$

Using trig identity:

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

we get $\lim_{h \to 0} \frac{(\cos x \cosh - \sin x \sin h) - \cos x}{h}$

factor sing out the $\cos x$ term, we get:

$\lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$

This can be split into 2 fractions:

$\lim_{h \to 0} \frac{\cos x(\cos h - 1)}{h} - \frac{\sin x \sin h}{h}$

$\lim_{h \to 0} \cos x \frac{(\cos h - 1)}{h} - \sin x \frac{(\sin h)}{h}$

note $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

Thus

$\lim_{h \to 0} \cos x(0) - \sin x(1)$

which equals $\lim_{h \to 0} (-\sin x$

8) Find dy/dx if $y = 3t^2$ and $x = 1/t^2$

Solution

$y = 3t^2 \quad \frac{dy}{dt} = 6t$

$x = \frac{1}{t^2} \quad -t = \frac{1}{\sqrt{x}}$

$dy/dx = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dt}{dx} = -\frac{1}{2\sqrt{x^3}}$

$\frac{dy}{dx} = 6t \times \left[-\frac{1}{2\sqrt{x^3}}\right]$

from $y = 3t^2$

$t = \sqrt{\frac{y}{3}}$

Thus $\frac{dy}{dx} = -6\sqrt{y/3} \times \frac{1}{2\sqrt{x^3}}$ //

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3) Express y and an explicit function of in x in the following
a) $2x-3y-2=0$
b) $x^2+y^2=4$

Solution

a) $2x-3y-2=0$

$2x-3y=2$

$2-3\frac{dy}{dx}=0$

$-3\frac{dy}{dx}=-2$

$\frac{dy}{dx}=\frac{-2}{-3}$

$=\frac{2}{3}$

b) $x^2+y^2=4$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-\cancel{2}x}{\cancel{2}y}$

$\frac{dy}{dx}=\frac{-x}{y}$

4) If $p=\sin^{-1}t$, find the derivative of P

Solution

Since $P=\sin^{-1}t$ then $\sin P=t$

Applying implicit differentiation we get;

$\cos(P)\frac{dp}{dt}=1$

$\frac{dp}{dt}=\frac{1}{\cos(P)}$ $\Rightarrow$ Now using trig. identity $\cos^2(P)+\sin^2(P)=1$

thus $\cos(P)=\sqrt{1-\sin^2(P)}$

$$\frac{dp}{dt}=\frac{1}{\sqrt{1-\sin^2(P)}}$$

recall $\sin(P)=t$

$$\frac{dp}{dt}=\frac{1}{\sqrt{1-t^2}}$$