3. Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

Solution:

1. $y - 3x - 2 = 0$; and $3y + x + 9 = 0$

Using $y = mx + c$

$y = 3x + 2$

$m_1 = 3$

$3y = -x - 9$

$y = \frac{-x}{3} - 3$

$m_2 = \frac{-1}{3}$

∴ Perpendicular lines $= (m_1 m_2) = -1$

$3 \times -1/3 = -1$ i.e. they are perpendicular to each othe

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Using $y = mx + c$ Using $y = mx + c$

$3y = 2x + 7$ $y = x + 11$

$m_2 = 1$

$y = \frac{2x}{3} + \frac{7}{3}$

$m_1 = \frac{2}{3}$

$\therefore$ Perpendicular lines $= (m_1 m_2) = -1$

$\frac{2}{3} \times 1 = 2/3$ ie they are not perpendicular to each

3. $x^2 + y^2 + 3xy - 11 = 0$ at $(1, 2)$

$2x + 2y\frac{dy}{dx} + 3\left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$\frac{dy}{dx}\left[2y + 3x\right] = -3y - 2x$

$\frac{dy}{dx} = \frac{-(3y + 2x)}{2y + 3x}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-(3(2) + 2(1))}{2(2) + 3(1)} = \frac{-8}{7}$

$m = -8/7$

$x_1 = 1, \; y_1 = 2$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-8}{7}(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x - \underline{\underline{22}} = 0$

Equation of normal

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 2 = \frac{7}{8}(x - 1)$

$8y - 16 = 7x - 7$

$8y - 7x - \underline{\underline{9}} = 0$