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(1)

These lines are
Examine whether or not these pair of perpendicular to each other

(i) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

(ii) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

(iii) find the equation of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at point $x = 1, y = 2$

Solutions

(i) $y - 3x - 2 = 0$

$3y + x + 9 = 0$

for the lines to be perpendicular then $m_1 m_2 = -1$:

$y - 3x - 2 = 0$

$y = 3x + 2$

By comparison with $y = mx + c$

$m_1 = 3$

$3y + x + 9 = 0$

$3y = -x - 9$

$y = \frac{-x}{3} - \frac{9}{3}$

$y = \frac{-1x}{3} - 3$

$y = mx + c \quad m_2 = -1/3$

$m_1 m_2 = -1$ for perpendicularity

$3 \times \frac{-1}{3} = -1$

Since $m_1 m_2 = -1$, therefore the lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular.

(1)

(2)

(2) $3y - 4 = 2x + 3$ - (1)

$y - 5 = x + 6$ - (2)

In equation (1)

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2x}{3} + \frac{7}{3}$

In comparison to $y = mc + x$

$m_1 = \frac{2}{3}$

In equation (2)

$y = x + 6 + 5$

$y = x + 11$

In comparison to $y = mc + x$

$m_2 = 1$

For perpendicularity $m_1 m_2 = -1$

$m_1 m_2 = \frac{2}{3} \times 1 = 2/3$

$m_1 m_2$ is not equal to -1 in the lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular

(3) x^2

m

x^2

$\frac{dy}{dx}$

m=

(a) Equati

y

y

(3) $x^2 + y^2 + 3xy - 4 = 0 \quad (x=1, y=2)$

$m = \frac{dy}{dx}$

$x^2 + y^2 + 3xy - 4 = 0$

$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$\frac{dy}{dx}(2y + 3x) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$

$m = \frac{dy}{dx}\Big|_{x=1, y=2} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$

(a) Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-8}{7}(x - 1)$

$7y - 14 = -8x + 8 \quad 7(y-2) = -8(x-1)$

$7y - 2 \qquad 7y - 14 = -8x + 8$

$7y + 8x - 14 + 8 = 0$

$7y + 8x - 22 = 0$ (Equation of the tangent

(b) Equation of the normal

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 2 = -1/-8/7 (x - 1)$

$y - 2 = \frac{7}{8}(x - 1)$

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$8(y-2) = 7(x-1)$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$

$8y - 7x - 9 = 0$ (Equation of the normal)