Examine whether or not these pair of lines are perpendicular to each other.

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 3$

(3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the points $(1, 2)$.

Solution

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

$y = mx + c$

$y = 3x + 2$ $\qquad 3y = -x - 9$

$m_1 = 3$ $\qquad m_2 = -1/3$

In an instance of 2 perpendicular lines:

$m_1 m_2 = -1$

$m_1 m_2 = \cancel{3} \times \frac{-1}{\cancel{3}} = -1$

$\therefore$ $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

$3y - 4 = 2x + 3$ and $y - 5 = x + 3$

$y = mx + c$

$3y = 2x + 7$ and $y = x + 8$

In an instance of perpendicular lines:

$m_1 = 2$; $m_2 = 1$

$m_1 m_2 = -1$

$2 \times 1 \neq -1$

$2 \times 1 = 2$

$\therefore$ $3y - 4 = 2x + 3$ and $y - 5 = x + 3$ are not perpendicular.

3 $\quad x^2 + y^2 + 3xy - 11 = 0$ at $(1, 2)$

differentiating by implicit method:

$$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y \cdot 1\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx}(2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

For equation of tangent:

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$m_1 = \frac{dy}{dx}\Big|_{x=x_1}$$

$$\frac{dy}{dx} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3}$$

$$\frac{dy}{dx} = \frac{-8}{7}$$

$$y-y_1 = m_1(x-x_1)$$

$$y-2 = \frac{-8}{7}(x-1)$$

$$7(y-2) = -8(x-1)$$

$$7y-14 = -8x+8$$

$\therefore$ The equation for tangent is: $7y+8x-22=0$

For equation of normal:

$$m_2 = \frac{-1}{m_1} = \frac{-1}{1} \times \frac{7}{-8} = \frac{7}{8}$$

$$y-y_1 = m_2(x-x_1)$$

$$y-2 = \frac{7}{8}(x-1)$$

$$8(y-2) = 7(x-1)$$

$$8y-16 = 7x-7$$

$\therefore$ Equation at normal is: $8y-7x-9=0$